A.P. and G.P.

sequence & series

- 1, 4, 9, 16, 25, 36, 49 .... It is a series of squares of Natural numbers
- 2, 3, 5, 7, 11, 13, 17, 23 .... It is a series of prime Numbers.

a, b, c, d, e, f, g, h, i are said to be in

Arithmetic progression (A.P.)

when $(b-a) = (c-b) = (d-c) = (e-d)$
$= (f-e) = (g-f) = (h-g) = (i-h)$
$= (\text{succ. term} - \text{preceding term})$
= common difference

Geometric progression (G.P.)

when $\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = \frac{e}{d}$
$= \frac{f}{e} = \frac{g}{f} = \frac{h}{g} = \frac{i}{h}$
$= \left(\frac{\text{succ. term}}{\text{preceding term}}\right)$
= common Ratio

- If (succeding term – preceding term) is same through out the series, It is called as A.P.
- If (succeding term / preceding term) is same through out the series, It is called as GP

Examples of A.P.

① 5, 11, 17, 23, 29, 35, 41 ....
② 30, 27, 24, 21, 18, 15, 12, 9, 6, 3, 0, −3, −6, −9
③ $31/3, 33/3, 35/3, 37/3, 39/3, 41/3$

④ 1.50, 3.25, 5, 6.75

Examples of G.P.

① 10, 20, 40, 80, 160, 320

② 8, 12, 18, 27

③ 10, −30, 90, −270, 810, −2430

④ $\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}$

⑤ $3, 3^2, 3^3, 3^4, 3^5, 3^6$ ......

Terms	AP / GP / Both of these / None of these
5, 10, 20, 30, 40, 80	Neither AP nor G.P.
20, 30, 45, 67.50	G.P. with $r = 1.50$ = common ratio
100, 90, 80, 70, 60, 50	A.P. with $d = -10$ = common Difference
$\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2, \frac{7}{3}, \frac{8}{3}$	A.P. with $d = \frac{1}{3}$
$300, 100, \frac{100}{3}$	G.P. with $r = \frac{1}{3} = 0.333333333$
28, 33, 38, 43, 58, 63	Neither AP nor G.P.
a, ax, ax^2, ax^3, ax^4	G.P. with $r = x$
8+m, 16+m, 24+m, 32+m	A.P. with $d = 8$
18, 18, 18, 18, 18, 18	Both AP as well as GP $d = 0$, $r = 1$
15, 155, 1555, 15555	Neither AP nor GP
1, 4, 9, 16, 25, 36, 49, 64	Neither AP nor GP

5, 55, 555, 5555	Neither AP nor G.P.
$x+3, x+8, x+13, x+18$	A.P. with $d=5$
$x, 3x, 5x, 7x, 9x, 11x$	A.P. with $d=2x$
$m, m^5, m^9, m^{13}, m^{17}$	G.P. with $r=m^4$
18, 11, 4, −3, −10, −17	A.P. with $d=-7$
99, 9.90, 0.99, 0.099	G.P. with $r=0.10=1/10$
4, −8, −16, 32, −64	Neither AP nor GP

- Terms can be in AP as well as GP if and only if all the terms are same
- a, b, c, d, e, f can be in AP as well GP if and only if $a=b=c=d=e=f$

Few Basic concepts

① $1+2+3 = 6 = \frac{3\times4}{2}$

$1+2+3+4+5+6+7 = 28 = \frac{7\times8}{2}$

$1+2+3+4+5+6+7+8+9+10+11+12 = 78 = \frac{12\times13}{2}$

$1+2+3+4+5+\cdots\cdots+1008 = ? = \frac{1008\times1009}{2}$

$= 5,08,536$

$1+2+3+4+5+\cdots\cdots+n = \left[\frac{n(n+1)}{2}\right]$

sum of first 'n' natural numbers = $\left[\frac{n(n+1)}{2}\right]$

② $1 + 4 + 9 + 16 = 30$

$1^2 + 2^2 + 3^2 + 4^2 = 30 = \left(\frac{4 \times 5 \times 9}{6}\right)$

$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 = 140 = \left(\frac{7 \times 8 \times 15}{6}\right)$

$1^2 + 2^2 + 3^2 + 4^2 + \text{------} + 12^2 = 650 = \left(\frac{12 \times 13 \times 25}{6}\right)$

$1^2 + 2^2 + 3^2 + \text{-------} + 833^2 = \left(\frac{833 \times 834 \times 1667}{6}\right)$

$= 193016929$

$1^2 + 2^2 + 3^2 + \text{------} + n^2 = \left[\frac{n(n+1)(2n+1)}{6}\right]$

sum of squares of first 'n' natural numbers	$= \frac{n(n+1)(2n+1)}{6}$

③ $1^3 + 2^3 + 3^3 = 36 = \left(\frac{3 \times 4}{2}\right)^2$

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 = 441 = \left(\frac{6 \times 7}{2}\right)^2$

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 + 10^3 = 3025$

$= \left(\frac{10 \times 11}{2}\right)^2$

$1^3 + 2^3 + 3^3 + \text{-----} + 90^3 = \left(\frac{90 \times 91}{2}\right)^2 = 16769025$

$1^3 + 2^3 + 3^3 + \text{------} + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

Sum of cubes of first 'n' natural numbers $= \left[\frac{n(n+1)}{2}\right]^2$

(4) $1+3+5 = 9 = 3^2$

$1+3+5+7+9 = 25 = 5^2$

$1+3+5+7+9+11+13+15+17+19+21+23 = 144 = 12^2$

$1+3+5+7+9+ \text{-------------} + 3999 = 2000^2$
$= 40,00,000$

$1+3+5+7+ \text{--------} + n \text{ terms} = (n)^2$

Sum of first 'n' odd natural numbers $= n^2$

(5) $2+4+6+8 = 20 = 4\times5$

$2+4+6+8+10+12 = 42 = 6\times7$

$2+4+6+8+10+12+14+16+18+20 = 110 = 10\times11$

Sum of first 'n' even natural numbers $= n(n+1)$

$2+4+6+8+10+ \text{-----} + n \text{ terms} = n(n+1)$
$= n^2+n$

(a) $1+2+3+4+ \text{--------} + n = n(n+1)/2$

(b) $1^2+2^2+3^2+4^2+5^2+ \text{------} + n^2 = n(n+1)(2n+1)/6$

(c) $1^3+2^3+3^3+4^3+ \text{--------} + n^3 = [n(n+1)/2]^2$

(d) $1+3+5+7+ \text{--------} + n \text{ terms} = n^2$

(e) $2+4+6+8+ \text{------} + n \text{ term} = n(n+1) = n^2+n$

① $25^2 + 26^2 + 27^2 + \cdots\cdots + 130^2 = ?$

$$\Rightarrow (1^2 + 2^2 + 3^2 + \cdots\cdots + 130^2) - (1^2 + 2^2 + \cdots\cdots + 24^2)$$

$$= \left(\frac{130 \times 131 \times 261}{6}\right) - \left(\frac{24 \times 25 \times 49}{6}\right)$$

$$= 7{,}35{,}905$$

② $30^3 + 31^3 + 32^3 + \cdots\cdots + 98^3 = ?$

$$\Rightarrow (1^3 + 2^3 + 3^3 + \cdots\cdots + 98^3) - (1^3 + 2^3 + 3^3 + \cdots\cdots + 29^3)$$

$$= \left(\frac{98 \times 99}{2}\right)^2 - \left(\frac{29 \times 30}{2}\right)^2$$

$$= 23342976$$

③ $85 + 86 + 87 + 88 + \cdots\cdots + 10580 = ?$

$$\Rightarrow (1 + 2 + 3 + \cdots\cdots + 10580) - (1 + 2 + 3 + \cdots\cdots + 84)$$

$$= \left(\frac{10580 \times 10581}{2}\right) - \left(\frac{84 \times 85}{2}\right)$$

$$= 55969920$$

8896, 8899, 8902, 8905 ....

In this A.P. First term = a = 8896 = t_1

common diff = d = 3

$t_1 = 8896 = a$

$t_2 = 8896 + 3 = a + d$

$t_3 = 8899 + 3 = (a+d) + d = a + 2d$

$t_4 = t_3 + d = a + 2d + d = a + 3d$

$t_5 = a + 4d$

$t_6 = a + 5d$

$t_{10} = a + 9d$

$t_{188} = a + 187d$

$t_n = a + (n-1)d$

n^{th} term of A.P. = $t_n = a + (n-1)d$

For AP: $t_m = a + (m-1)d$

$t_n = t_{n-1} + d = t_{n+1} - d$

$t_{100} = t_{101} - d = t_{99} + d$

$t_{188} = t_{187} + d = t_{189} - d$

$t_{200} + 2d = t_{202}$

$t_{19} + 5d = t_{24}$

In AP: Any term can be obtained by adding common diff to its preceding term OR By deducting common diff from its succeding term.

$\therefore t_p = t_{p-1} + d$

$= t_{p+1} - d$

① 89, 93, 97, 101, 105 .... Find t_{100}, t_{205}, t_{1083}

⟹ In this A.P. $a = 89$, $d = 4$

$t_{100} = a + 99d = 89 + (99 \times 4) = 485$

$t_{205} = a + 204d = 89 + (204 \times 4) = 905$

$t_{1083} = a + 1082d = 89 + (1082 \times 4) = 4417$

② 587, 593, 599, 605, ... Find t_{190}, t_{810}, t_{59}

⟹ In this A.P. $a = 587$, $d = 6$

$t_{190} = a + 189d = 587 + (189 \times 6) = 1721$

$t_{810} = a + 809d = 587 + (809 \times 6) = 5{,}441$

$t_{59} = a + 58d = 587 + (58 \times 6) = 935$

③ 110, 107, 104, 101, ... Find t_{40}, t_{800}, t_{555}

⟹ In this A.P. $a = 110$, $d = -3$

$t_{40} = a + 39d = 110 + (39 \times -3) = -7$

$t_{800} = a + 799d = 110 + (799 \times -3) = -2287$

$t_{555} = a + 554d = 110 + (554 \times -3) = -1552$

④ 1.85, 1.91, 1.97, 2.03, 2.09, ... Find t_{1000}, t_{888}

⟹ In this A.P. $a = 1.85$, $d = 0.06$

$t_{1000} = a + 999d = 1.85 + (999 \times 0.06) = 61.79$

$t_{888} = a + 887d = 1.85 + (887 \times 0.06) = 55.07$

⑤ 5888, 5892, 5896, 5900, ..., 10100

How many terms are there in above AP?

⟹ In this A.P. $a = 5888$, $d = 4$, $t_n = 10{,}100$, $n = ?$

$t_n = a + (n-1)d$

$10100 = 5888 + (n-1)4$

$10{,}100 - 5888 = (n-1)4$

$$4212 = (n-1)4$$

$$\therefore \frac{4212}{4} = n-1$$

$$1053 = n-1$$

$$\therefore n-1 = 1053 \quad \text{---- changing the sides}$$

$$n = 1053+1$$

$$n = 1054$$

$\therefore$ There are 1054 terms in above A.P.

⑥ 899, 904, 909, 914, ----------, 1284344

How many terms are there in above A.P.

$\Rightarrow$ In this AP $a = 899$, $d = 5$

$$t_n = a + (n-1)d$$

$$1284344 = 899 + (n-1)5$$

$$n = 2,56,690$$

There are 2,56,690 terms in above A.P.

⑦ $89^3 + 90^3 + 91^3 + \text{-----} + 110^3 = ?$

$$\Rightarrow (1^3 + 2^3 + 3^3 + \text{-----} + 110^3) - (1^3 + 2^3 + 3^3 + \text{-----} + 88^3)$$

$$= \left(\frac{110 \times 111}{2}\right)^2 - \left(\frac{88 \times 89}{2}\right)^2$$

$$= 21935969$$

⑧ $59^2 + 60^2 + 61^2 + 62^2 + \text{-----} + 190^2 = ?$

$$\Rightarrow (1^2 + 2^2 + 3^2 + \text{------} + 190^2) - (1^2 + 2^2 + 3^2 + \text{-----} + 58^2)$$

$$= \left(\frac{190 \times 191 \times 381}{6}\right) - \left(\frac{58 \times 59 \times 117}{6}\right)$$

$$= 22,37,686$$

(9) $588 + 593 + 598 + 603 + \cdots\cdots$ Find sum of 2000 terms $= S_{2000} = ?$

$\Longrightarrow$ Let's first derive formula of S_n = sum of first 'n' terms of AP

$$S_n = t_1 + t_2 + t_3 + t_4 + \cdots\cdots + t_n$$

$$= a + (a+d) + (a+2d) + (a+3d) + \cdots\cdots + [a+(n-1)d]$$

$$S_n = (a + a + a + a + \cdots\cdots n \text{ terms}) + (d + 2d + 3d + \cdots (n-1) \text{ terms})$$

$$S_n = n.a + d[1 + 2 + 3 + 4 + \cdots\cdots + (n-1)]$$

$$S_n = n.a + d \times \frac{(n-1)(n-1+1)}{2}$$

$$S_n = n.a + d \times \frac{n(n-1)}{2}$$

$$S_n = \left[\frac{2n.a}{2} + \frac{n(n-1)d}{2}\right] = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_n = \frac{n}{2}\left[a + a + (n-1)d\right]$$

$$= \frac{n}{2}\left[a + t_n\right] = \frac{n}{2}\left[t_1 + t_n\right]$$

$$S_{2000} = \frac{2000}{2}\left[2a + 1999d\right]$$

$$= 1000\left[1176 + (1999 \times 5)\right]$$

$$= 11171000/-$$

For AP

$$t_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

OR

$$S_n = \frac{n}{2}\left[t_1 + t_n\right]$$

(10) 577, 585, 593, 601 .... Find t_{560}, t_{890}, S_{501}, S_{888}, S_{1000}

$\Longrightarrow$ In this A.P. $a = 577$, $d = 8$

i) $t_{560} = a + 559d = 577 + (559 \times 8) = 5049$

ii) $t_{890} = a + 889d = 577 + (889 \times 8) = 7689$

iii) $S_{501} = \frac{501}{2}\left[1154 + (500 \times 8)\right] = 12,91,077$

iv) $S_{888} = \frac{888}{2}\left[1154 + (887 \times 8)\right] = 36,63,000$

v) $S_{1000} = \frac{1000}{2}\left[1154 + (999 \times 8)\right] = 45,73,000$

(11) $5888 + 5892 + 5896 + \ldots\ldots + 6900 = ?$

$\Longrightarrow$ $t_n = a + (n-1)d$

$6900 = 5888 + (n-1)4$

$n = 254$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ OR $S_n = \frac{n}{2}(t_1 + t_n)$

$S_{254} = \frac{254}{2}\left[11,776 + (253 \times 4)\right]$

$= 16,24,076$

$S_{254} = \frac{254}{2}(t_1 + t_{254})$

$= \frac{254}{2}(5888 + 6900)$

$= 16,24,076$

(12) $2887 + 2893 + 2899 + \ldots\ldots + 6415 = ?$

$\Longrightarrow$ $6415 = 2887 + (n-1)6$

$n = 589$

$S_{589} = \frac{589}{2}(2887 + 6415)$

$= 27,39,439$

OR

S_{589}

$= \frac{589}{2}\left[5774 + (588 \times 6)\right]$

$= 27,39,439$

(13) $1083 + 1090 + 1097 + 1104 + \text{-------} + 89066 = ?$

$\Longrightarrow$ $89066 = 1083 + (n-1)7$

$n = 12,570$

$$S_{12570} = \frac{12570}{2}\left(1083 + 89066\right)$$

$= 566586465$

(14) $9999 + 10,004 + 10,009 + \text{------}$ Find S_{80}, S_{122}

$\Longrightarrow$

$$S_{80} = \frac{80}{2}\left[19998 + (79 \times 5)\right] = 815720$$

$$S_{122} = \frac{122}{2}\left[19998 + (121 \times 5)\right] = 12,56,783$$

(15) Find sum of all natural numbers divisible by 7 between 10,000 and 2,00,000

$\Longrightarrow$ $10,003 + 10,010 + 10,017 + \text{--------} + 1,99,997 = ?$

$1,99,997 = 10003 + (n-1)7$

$n = 27143$

$$S_{27143} = \frac{27143}{2}\left(10003 + 199997\right)$$

$= 2850015000$

(16) Find sum of all 4 digit numbers divisible by 9

$\Longrightarrow$ $1008 + 1017 + 1026 + \text{-------} + 9999 = ?$

$\therefore$ $9999 = 1008 + (n-1)9$

$n = 1000$

$$S_{1000} = \frac{1000}{2}(1008 + 9999) = 5503500$$

(17) Find sum of all natural numbers divisible by 13 between 1000 and 90,000

$\Rightarrow$ $1001 + 1014 + 1027 + \text{------} + 89{,}999 = ?$

$89999 = 1001 + (n-1)13$

$n = 6847$

$S_{6847} = \frac{6847}{2}(1001 + 89999)$

$= 311538500$

(18) Find sum of all natural numbers divisible by 17 between 11,000 and 1,20,000

$\Rightarrow$ $11{,}016 + 11033 + 11050 + \text{------} + 1{,}19{,}986 = ?$

$119986 = 11016 + (n-1)17$

$n = 6411$

$S_{6411} = \frac{6411}{2}(11016 + 119986)$

$= 419926911$

(19) Find sum of all natural numbers divisible by 5 from 100 to 900.

$\Rightarrow$ $100 + 105 + 110 + \text{------} + 900 = ?$

$900 = 100 + (n-1)5$

$n = 161$

$S_{161} = \frac{161}{2}(100 + 900) = 80{,}500$

(20) Find sum of all natural numbers between 200 and 2500 such that on division by 12 number leaves remainder of 5.

$\Rightarrow$ $209 + 221 + 233 + \text{------} + 2489 = ?$

$2489 = 209 + (n-1)12$

$n = 191$

$S_{191} = \frac{191}{2}(209 + 2489) = 257659$

(21) Find sum of all such numbers between 100 and 800 such that on division by 16 that number leaves remainder of 7.

$\Longrightarrow$ $103 + 119 + 135 + \text{-------} + 791 = ?$

$791 = 103 + (n-1)16$

$n = 44$

$S_{44} = \frac{44}{2}(103 + 791) = 19668$

(22) Find sum of all numbers between 1000 and 12,000 such that on division by 10 that number leaves remainder of 8.

$\Longrightarrow$ $1008 + 1018 + 1028 + \text{--------} + 11,998 = ?$

$11,998 = 1008 + (n-1)10$

$n = 1100$

$S_{1100} = \frac{1100}{2}(1008 + 11998) = 7153300$

(23) For AP $t_8 = 238$, $t_{13} = 327$ Find t_{30}, t_{90}, S_{80}

$\Longrightarrow$

$\cancel{a} + 12d = 327$

$\cancel{a} + 7d = 238$

$- \quad - \quad -$

$5d = 89$

$\boxed{d = 17.80}$

$a + (12 \times 17.80) = 327$

$\boxed{a = 113.40}$

$t_{30} = a + 29d$

$= 113.40 + (29 \times 17.80)$

$= 629.60$

$t_{90} = a + 89d$

$= 113.40 + (89 \times 17.80)$

$= 1697.60$

$S_{80} = \frac{80}{2}\boxed{226.80 + (79 \times 17.80)}$

$= 65,320$

(24) For A.P. $t_{80} = 5087$
$t_{92} = 5843$

Find t_{200}, S_{85}

$\Longrightarrow$

$$\cancel{a} + 79d = 5087$$
$$\cancel{a} + 91d = 5843$$
$$- \quad - \qquad -$$
$$-12d = -756$$
$$\boxed{d = 63}$$
$$a + (79 \times 63) = 5087$$
$$\boxed{a = 110}$$

$t_{200} = 110 + (199 \times 63)$
$= 12647$

S_{85}
$= \frac{85}{2}\left[220 + (84 \times 63)\right]$
$= 234260$

(25) For AP $t_{80} = 53810$
$t_{110} = 65850$

Find a, d, t_{200}, S_{250}

$\Longrightarrow$

$$\cancel{a} + 79d = 53810$$
$$\cancel{a} + 109d = 65850$$
$$- \quad - \qquad -$$
$$-30d = -12040$$
$$\boxed{d = 401.33333}$$
$$a + (79 \times 401.333333) = 53810$$
$$\boxed{a = 22104.66666}$$

t_{200}
$= a + 199d$
$= 22104.6666 + (199 \times 401.33333)$
$= 1,01,970/-$

S_{250}
$= \frac{250}{2}\left[44209.33333 + (249 \times 401.333)\right]$
$= 180\ 17666.66666$

26. For AP $t_8 = 90$

$S_{20} = 1080$

Find a, d, t_{10}, S_{40}

$\Longrightarrow$

$a + 7d = 90$ ----- ①

$S_{20} = 10(2a + 19d) = 1080$

$2a + 19d = 108$ ---- ②

$\cancel{2a} + 19d = 108$

$-\cancel{2a} \pm 14d = -180$

$5d = -72$

$\boxed{d = -14.40}$

$a + (7 \times -14.40) = 90$

$\boxed{a = 190.80}$

$t_{10} = 190.80 + (9 \times -14.40)$

$= 61.20$

$S_{40} = \frac{40}{2}\left[381.60 + (39 \times -14.40)\right]$

$= -3600$

27. For AP $t_m = n$, $t_n = m$ Find $t_r = ?$

(a) zero (b) $m + n + 2r$ (c) $m + n - r$ (d) $2m + n - 2r$

$\Longrightarrow$ $t_m = n$

$a + (m-1)d = n$

$a + md - d = n$ --- ①

$t_n = m$

$a + (n-1)d = m$

$a + nd - d = m$ ----- ②

solving eqns ① & ②

$\cancel{a} + md - \cancel{d} = n$

$-\cancel{a} \pm nd \mp \cancel{d} = -m$

$md - nd = n - m$

$d(\cancel{m-n}) = -1(\cancel{m-n})$

$\boxed{d = -1}$

$a + md - d = n$

$a + m(-1) - (-1) = n$

$a - m + 1 = n$

$\boxed{a = m + n - 1}$

$t_r = a + (r-1)d$

$= m + n - 1 + (r-1)(-1)$

$= m + n - \cancel{1} - r + \cancel{1}$

$= m + n - r$

(28) For AP $S_n = (8n^2 + 10n)$ Find t_{25}.

$\Longrightarrow$

$S_n = 8n^2 + 10n$

$= n(8n+10)$

$= \frac{n}{2} \times 2 \times (8n+10)$

$= \frac{n}{2}(16n+20)$

$= \frac{n}{2}\left[16(n-1) + 16 + 20\right]$

$= \frac{n}{2}\left[36 + (n-1)16\right]$

$= \frac{n}{2}\left[2(18) + (n-1)16\right]$

$\therefore a = 18 \quad d = 16$

$t_{25} = a + 24d$

$= 18 + (24 \times 16)$

$= 402$

$S_n = 8n^2 + 10n$

$S_1 = 8(1)^2 + 10(1)$

$= 18$

$a = 18$

$S_2 = 8(2)^2 + 10(2)$

$= 52$

$a = 18,\ t_2 = 34,\ d = 16$

$t_{25} = a + 24d$

$= 18 + (24 \times 16)$

$= 402$

t_{25}

$= S_{25} - S_{24}$

$= \left[8(25)^2 + 10(25)\right] - \left[8(24)^2 + 10(24)\right]$

$= 5250 - 4848$

$= 402$

(29) For AP If $S_n = 10n^2 - 3n$ Find t_n, t_{50}

$\Longrightarrow$ $S_n = 10n^2 - 3n$

$S_1 = 10(1)^2 - 3(1) = 7$

$S_2 = 10(2)^2 - 3(2) = 34$

$a = 7,\ t_2 = 27,\ d = 20$

$t_n = 7 + (n-1)20$

$t_n = 7 + 20n - 20$

$t_n = 20n - 13$

$t_{50} = 20(50) - 13$

$= 987$

(30) For AP $S_n = 16n^2 - 5n$ Find t_n, t_{80}, t_{120}

$\Longrightarrow$ $S_n = 16n^2 - 5n$

$S_1 = 16(1)^2 - 5(1) = 11$

$S_2 = 16(2)^2 - 5(2) = 54$

$\therefore$ $a = 11, \; t_2 = 43, \; d = 32$

$t_n = a + (n-1)d = 11 + (n-1)32 = 11 + 32n - 32$

$= 32n - 21$

$t_{80} = 32(80) - 21 = 2539$

$t_{120} = 32(120) - 21 = 3819$

(31) For A.P. If $t_n = 16n - 9$
Find S_n, S_{20}, S_{28}

$\Longrightarrow$ $t_n = 16n - 9$

$t_1 = 16(1) - 9 = 7$

$S_n = \frac{n}{2}\left[t_1 + t_n\right] = \frac{n}{2}\left[7 + 16n - 9\right] = \frac{n}{2}(16n - 2)$

$= n(8n - 1) = 8n^2 - n$

$S_{20} = 8(20)^2 - 20 = 3180$

$S_{28} = 8(28)^2 - 28 = 6244$

(32) For AP If $t_n = \left(\frac{6n+11}{2}\right)$

Find S_n

$\Rightarrow$ $t_n = \frac{6n+11}{2}$ $\quad \therefore t_1 = \frac{6(1)+11}{2} = \frac{17}{2}$

$$S_n = \frac{n}{2}(t_1 + t_n) = \frac{n}{2}\left[\frac{17}{2} + \frac{6n+11}{2}\right]$$

$$= \frac{n}{2}\left[\frac{17+6n+11}{2}\right] = \frac{n}{2}\left(\frac{6n+28}{2}\right)$$

$$= \frac{n}{2}(3n+14) = \left(\frac{3n^2+14n}{2}\right) = (1.50n^2 + 7n)$$

(33) For AP $t_n = (6n-7)$ Find S_m

$\Rightarrow$ $t_n = 6n-7$

$t_1 = 6(1)-7 = -1$

$t_m = 6m-7$

$$S_m = \frac{m}{2}(t_1 + t_m)$$

$$= \frac{m}{2}(-1 + 6m - 7) = \frac{m}{2}(6m-8) = m(3m-4)$$

$$S_m = (3m^2 - 4m)$$

(34) For AP $S_m = \left(\frac{8m^2+5m}{3}\right)$ Find t_n

$\Rightarrow$

$$S_m = \left(\frac{8m^2+5m}{3}\right)$$

$$S_1 = \frac{8(1)^2+5(1)}{3} = \frac{13}{3} \qquad a = \frac{13}{3}$$

$$S_2 = \frac{8(2)^2+5(2)}{3} = \frac{42}{3} \qquad t_2 = \frac{29}{3} \qquad d = \frac{16}{3}$$

$$t_n = a+(n-1)d = \frac{13}{3} + (n-1)\frac{16}{3} = \frac{13}{3} + \frac{16n}{3} - \frac{16}{3}$$

$$= \frac{16n}{3} - \frac{3}{3} = \left(\frac{16n-3}{3}\right) = \left(\frac{16n}{3} - 1\right)$$

(35) For AP $S_n = (12n^2 - 2n)$ Find t_{35}

$\Rightarrow$ $S_n = 12n^2 - 2n$

$S_1 = 12(1)^2 - 2(1) = 10$ $a = 10$

$S_2 = 12(2)^2 - 2(2) = 44$ $t_2 = 34$, $d = 24$

$t_{35} = a + 34d$

$= 10 + 34(24) = 826$

(36) For AP $t_{95} = 2083.50$

$t_{105} = 5811.25$

Find a, d, t_{200}, S_{558}

$\Rightarrow$ $\cancel{a} + 94d = 2083.50$

$\cancel{a} + 104d = 5811.25$

$- \quad - \quad -$

$-10d = -3727.75$

$\boxed{d = 372.775}$

$a + 94(372.775) = 2083.50$

$\boxed{a = -32957.35}$

$t_{200} = -32957.35 + (199 \times 372.775)$

$= 41,224.875$

$S_{558} = \frac{558}{2}\left[-65914.70 + (557 \times 372.775)\right]$

$= 39540152.025$

(37) 9^{th} term of AP is 93 and 93^{rd} term of AP is 9. Find 102^{nd} term of AP.

$\Rightarrow$ $\cancel{a} + 8d = 93$

$\_\cancel{a} \pm 92d = \_9$

$-84d = 84$

$d = -1$

$a + 8(-1) = 93$

$a - 8 = 93$

$a = 101$

If $t_m = n$ & $t_n = m$ then $t_{(m+n)} = 0$

$t_9 = 93$, $t_{93} = 9$, then $t_{93+9} = t_{102} = 0$

$\Uparrow$ For AP

For AP If p^{th} term is q and q^{th} term is p then

$t_{(p+q)}$ = zero

(38) For AP If $t_{90} = 120$, $t_{120} = 90$

Find t_{100}

$\Longrightarrow$

$$\cancel{a} + 89d = 120$$
$$\underline{-\cancel{a} + 119d = -90}$$
$$-30d = 30$$
$$d = -1$$
$$a + 89(-1) = 120$$
$$a = 209$$

t_{100}

$= a + 99d$

$= 209 + 99(-1)$

$= 110$

If possible please remember,

For AP If $t_m = n$, $t_n = m$ then $t_p = m + n - p$

(39) For AP If $10 \times t_{10} = 30 \times t_{30}$

Find t_{40}

$\Longrightarrow$

$$10 \times t_{10} = 30 \times t_{30}$$
$$10(a + 9d) = 30(a + 29d)$$
$$10a + 90d = 30a + 870d$$
$$-780d = 20a$$
$$\boxed{a = -39d}$$

t_{40}

$= a + 39d$

$= -39d + 39d$

$= 0$

For AP If $m \times t_m = n \times t_n$ then

$t_{(m+n)}$ = zero = 0

(40) For AP If $t_{60} \times 60 = 200 \times t_{200}$

Find t_{260}

$\Longrightarrow$ $60(a+59d) = 200(a+199d)$

$60a + 3540d = 200a + 39800d$

$-36260d = 140a$

$\boxed{a = -259d}$

$\therefore\ t_{260} = a + 259d$

$= -259d + 259d$

$= 0$

(41) For AP If $S_{30} = S_{50}$ Find S_{80}

$\Longrightarrow$

$S_{30} = S_{50}$

$\frac{30}{2}(2a+29d) = \frac{50}{2}(2a+49d)$

$30a + 435d = 50a + 1225d$

$-790d = 20a$

$\boxed{-79d = 2a}$

S_{80}

$= \frac{80}{2}[2a + 79d]$

$= 40(2a+79d)$

$= 40(-79d+79d)$

$= 40 \times 0$

$= 0$ = zero

For A.P. If $S_m = S_n$ then $S_{(m+n)} = 0$

For A.P.

① If $t_p = q$, $t_q = p$ then $t_{p+q} = 0$

② If $p \times t_p = q \times t_q$ then $t_{p+q} = 0$

③ If $S_p = S_q$ then $S_{(p+q)} = 0$

For AP

① If m^{th} term is 'n' & n^{th} term if 'm' then $(m+n)^{th}$ term must be zero

② If 'm' times of m^{th} term is equal to 'n' times of n^{th} term then $(m+n)^{th}$ term must be zero

③ If sum of first 'm' terms of AP is equal to sum of first 'n' terms of AP then sum of first 'm+n' terms must be zero

(42) $(3x+7), (5x+8), (19x-17)$ are in A.P. Find x

$\Longrightarrow$

$$t_2 - t_1 = t_3 - t_2$$
$$(5x+8)-(3x+7) = (19x-17)-(5x+8)$$
$$5x+8-3x-7 = 19x-17-5x-8$$
$$2x+1 = 14x-25$$
$$26 = 12x$$
$$\therefore\ x = 26/12 = 13/6$$

(43) $(8k+7), (19k-8), (22k+38)$ are in A.P. Find value of k.

$\Longrightarrow$

$$19k-8-8k-7 = 22k+38-19k+8$$
$$11k-15 = 3k+46$$
$$8k = 61$$
$$k = 61/8 = 7.625$$

(44) $(3p-19), (8p-55), (20p+281)$ are in A.P. Find value of p.

$\Longrightarrow$

$$8p-55-3p+19 = 20p+281-8p+55$$
$$5p-36 = 12p+336$$
$$-372 = 7p$$
$$p = (-372/7) = -53.142857$$

45) $(2k-19), (-5k-63), (-9k+25)$ are in A.P. Find value of k.

$\Longrightarrow$ $-5k-63-2k+19 = -9k+25+5k+63$

$-7k-44 = -4k+88$

$-3k = 132$

$k = \left(-132/3\right) = -44$

46) If $\log a, \log b, \log c$ are in A.P. then a, b, c are in ______

(a) A.P. ~~(b)~~ G.P. (c) H.P. (d) None of these

$\Longrightarrow$ $\log a, \log b, \log c$ are in A.P.

$\therefore \log b - \log a = \log c - \log b$

$\log\left(b/a\right) = \log\left(c/b\right)$

$\frac{b}{a} = \frac{c}{b}$

$\therefore$ a, b, c are in G.P.

x, y, z are said to be in

A.P. : $(y-x) = (z-y)$

G.P. If $\frac{y}{x} = \frac{z}{y}$

47) For AP $a = 48$, $t_n = 500$, $n = 30$

Find S_{30}

$\Longrightarrow$ $S_{30} = \frac{30}{2}\left(t_1 + t_{30}\right) = 15\left(48+500\right)$

$= 8220$

(48) $10 + 9\frac{2}{3} + 9\frac{1}{3} + 9 + 8\frac{2}{3} + 8\frac{1}{3} + \text{------}$

Find S_{30}

$\Rightarrow$

$\frac{30}{3} + \frac{29}{3} + \frac{28}{3} + \frac{27}{3} + \frac{26}{3} + \text{------}$

In this A.P. $a = \frac{30}{3} = 10$, $d = -\frac{1}{3}$

$S_{30} = \frac{30}{2}\left[2a + 29d\right]$

$= 15\left[20 + 29\left(-\frac{1}{3}\right)\right] = 15\left[\frac{60}{3} - \frac{29}{3}\right]$

$= 15 \times \frac{31}{3} = 155$

(49) Find sum of all 4 digit natural even numbers divisible by 9.

$\Rightarrow$ $1008 + 1026 + 1044 + \text{----------} + 9990 = ?$

$t_n = a + (n-1)d$

$9990 = 1008 + (n-1)18$

$n = 500$

$S_{500} = \frac{500}{2}(1008 + 9990)$

$= 27{,}49{,}500$

(50) Find sum of all 3 digit natural numbers divisible by 7.

$\Rightarrow$ $105 + 112 + 119 + \text{------} + 994 = ?$

$t_n = a + (n-1)d$

$994 = 105 + (n-1)7$

$\therefore n = 128$

$\therefore S_{128} = \frac{128}{2}(105 + 994)$

$= 70{,}336$

(51) $81^3 + 82^3 + 83^3 + \text{-----} + 120^3 = ?$

$\Rightarrow (1^3 + 2^3 + 3^3 + \text{-----} + 120^3) - (1^3 + 2^3 + 3^3 + \text{----} + 80^3)$

$= \left(\frac{120 \times 121}{2}\right)^2 - \left(\frac{80 \times 81}{2}\right)^2$

$= 4,22,10,000$

(52) $500^2 + 501^2 + 502^2 + \text{------} + 610^2 = ?$

$\Rightarrow (1^2 + 2^2 + 3^2 + \text{-----} + 610^2) - (1^2 + 2^2 + 3^2 + \text{----} + 499^2)$

$= \left(\frac{610 \times 611 \times 1221}{6}\right) - \left(\frac{499 \times 500 \times 999}{6}\right)$

$= 34304735$

(53) $9999 + 10,004 + 10009 + 10014 + \text{----} + 1192944 = ?$

$\Rightarrow$ In this A.P. $a = 9999$, $d = 5$

$t_n = a + (n-1)d$

$1192944 = 9999 + (n-1)5$

$n = 236590$

$S_{236590} = \frac{236590}{2}(9999 + 1192944)$

$= 142302142185$

(54) For AP $S_n = \left(\frac{11n^2 - 7n}{2}\right)$ Find t_n.

$\Rightarrow S_n = \left(\frac{11n^2 - 7n}{2}\right) \quad \therefore S_1 = 2. \quad \therefore a = 2$

$S_2 = \frac{11(2)^2 - 7(2)}{2} = 15 \quad t_2 = 13, \; d = 11$

$t_n = a + (n-1)d$

$= 2 + (n-1)11 = 2 + 11n - 11 = 11n - 9$

(55) If a, b, c are in A.P.

then 'b' is Arithmetic Mean (A.M.) of a & c

As a, b, c are in A.P.

$$b-a=c-b$$

$$b+b=a+c$$

$$2b=a+c$$

$$b=\left(\frac{a+c}{2}\right)$$

AM of a & c $=\left(\frac{a+c}{2}\right)$

AM of x & y $=\left(\frac{x+y}{2}\right)$

AM of m, n $=\left(\frac{m+n}{2}\right)$

AM of 50 & 150 $=\left(\frac{50+150}{2}\right)=100$

(56) ① If a, b, c, d are in A.P. then we can say that: b, c are 2 A.Means between a & d

- 57, 62, 67, 72 are in A.P.

∴ 62, 67 are 2 Arithmetic means between 57, 72

② If p, q, r, s are in A.P. then we can say that q, r are 2 A.means between p & s.

③ If p, q, r, s, t are in A.P. then we can say that q, r, s are 3 A.means between p & t

(4) 90, 101, 112, 123, 134, 145 are in A.P.

$\therefore$ 101, 112, 123, 134 are 4 A.means between 90 & 145

(57) 4, 10, 16, 22, 28, 34, 40, 46, 52, 58, 64, 70, 76, 82, 88, 94, 100, 106, 112, 118, 124, 130, 136, 142, 148, 154 are in A.P. with $d = 6$

4, 16, 28, 40, 52, 64, 76, 88, 100, 112, 124, 136, 148 are also in A.P. $d = 12$

4, 22, 40, 58, 76, 94, 112, 130, 148 are also in A.P. with $d = 18$

If a, b, c, d, e, f, g, h, i, j, k are in AP then

$$\left.\begin{matrix} a, c, e, g, i, k \\ a, d, g, j \\ a, e, i \end{matrix}\right\}\text{ are also in A.P.}$$

If a, b, c, d, e are in A.P. then

1. b, c, d are 3 A.means between a & e
2. b, c are 2 A.means between a & d
3. c, d are 2 A means between b & e
4. b is AM of a & c
5. c is AM of b & d
6. c is AM of a & e

(58) Find AM of 20 & 50

$$\implies AM = \left(\frac{20+50}{2}\right) = \frac{70}{2} = 35$$

20, 35, 50 are in A.P.

(59) Insert 2 A.means between 80 & 120

$\Longrightarrow$ 80, ◯, ◯, 120

$a = 80$, $t_4 = 120$

$a + 3d = 120$

$80 + 3d = 120$

$3d = 40$

$d = 13.333333$

80, 93.3333333, 106.66666666, 120 are in A.P.

$\therefore$ $93\frac{1}{3}$, $106\frac{2}{3}$ are 2 A.means betn 80 & 120

(60) Insert 5 A.means between 20 & 200

$\Longrightarrow$ 20, ◯, ◯, ◯, ◯, ◯, 200

$a = 20$

$t_7 = 200 = a + 6d$

$200 = 20 + 6d$

$180 = 6d$

$d = 30$

20, 50, 80, 110, 140, 170, 200 $\Longrightarrow$ This is AP of 7 terms

$\therefore$ 50, 80, 110, 140, 170 are 5 A.means betn 20 & 200

(61) Insert 7 A.means between 5000 & 80,000

$\Longrightarrow$ $a = 5000$, $t_9 = 80{,}000$

$a + 8d = 80{,}000$

$5000 + 8d = 80{,}000$

$\boxed{d = 9375}$

$\therefore$ 7 A.Means are : 14,375, 23,750, 33,125, 42,500, 51,875, 61,250, 70625

62 Insert 4 A.means between (−8256) and 5820.

$\Longrightarrow$ $a = -8256$

$t_6 = 5820$

$a + 5d = 5820$

$-8256 + 5d = 5820$

$5d = 14076$

$d = 2815.20$

$\therefore$ 4 A.Means are : −5440.80, −2625.60, 189.60, 3004.80

63 Insert 7 A.Means between 80 & 200

$\Longrightarrow$ $a = 80$

$t_9 = 200$

$a + 8d = 200$

$80 + 8d = 200$

$\boxed{d = 15}$

$\therefore$ 7 A.Means are : 95, 110, 125, 140, 155, 170, 185

80, $\boxed{95, 110, 125, 140, 155, 170, 185}$, 200

64 Insert 9 A.Means betn 208.60 & 4286.20

$\Longrightarrow$ $a = 208.60$

$t_{11} = a + 10d = 4286.20$

$208.60 + 10d = 4286.20$

$\boxed{d = 407.76}$

$\therefore$ 9 A.Means are :

616.36, 1024.12, 1431.88, 1839.64, 2247.40, 2655.16, 3062.92, 3470.68, 3878.44

65 Insert 11 A.Means between 2860 and 9640

$\Longrightarrow$ $a = 2860$ $\quad t_{13} = 9640$

$a + 12d = 9640$

$2860 + 12d = 9640$

$d = 565$

$\therefore$ 11 A.means are : 3425, 3990, 4555, 5120, 5685, 6250, 6815, 7380, 7945, 8510, 9075

66 Insert 2 A.Means betn -20 & 22

$\Longrightarrow$ $a = -20$ $\quad t_4 = 22$

$a + 3d = 22$

$-20 + 3d = 22$

$3d = 42$

$d = 14$

$\therefore$ 2 A.means are $-6, 8$

$-20, -6, 8, 22$ are in A.P.

67

2, 8, 32, 128, 512, 2048, 8192 -------- are in G.P.

Find t_8

$\Downarrow$

In this G.P. First term $= 2 = a$

second term $= t_2 = a \cdot r$

$t_3 = t_2 \cdot r = a \cdot r \cdot r = a \cdot r^2$

$t_4 = t_3 \cdot r = a \cdot r^2 \cdot r = a \cdot r^3$

$t_{10} = a \cdot r^9$

$t_{2000} = a \cdot (r)^{1999}$

$\therefore t_n = a \times (r)^{n-1}$

$\therefore$ n^{th} term of G.P. $= t_n = a \times (r)^{n-1}$
$= a \cdot r^{n-1}$

$t_8 = a \cdot r^7 = 2 \times (4)^7 = 32768$

(68) 5, 15, 45, 135 ------ Find t_{10}, t_{12}, t_{22}

$\Longrightarrow$ In this G.P. $a = 5$, $r = 3$

$t_{10} = a \times (r)^9 = 5 \times (3)^9 = 98,415$
$t_{12} = a \times (r)^{11} = 5 \times (3)^{11} = 8,85,735$
$t_{22} = a \times (r)^{21} = 5 \times (3)^{21} = 52301766015$

(69) 2, 3, 4.50, 6.75, ------ Find t_{23}, t_{28}, t_{40}

$\Longrightarrow$ In this G.P. $a = 2$, $r = 1.50$

$t_{23} = a \times (r)^{22} = 2 \times (1.50)^{22} = 14963.6552851$
$t_{28} = a \times (r)^{27} = 2 \times (1.50)^{27} = 113630.25732$
$t_{40} = a \times (r)^{39} = 2 \times (1.50)^{39} = 14743109.7607$

(70) $2 + 6 + 18 + 54 + 162 = 242$

by calculator

Let's solve by formula,

S_n For G.P. $= \boxed{\dfrac{a(r^n - 1)}{r - 1}}$ when $r > 1$

$$S_5 = \frac{2(3^5 - 1)}{(3-1)} = \frac{2 \times (243 - 1)}{2} = 242$$

(71) $10 + 15 + 22.50 + 33.75 + \cdots$ Find S_{20}

$\Rightarrow$ In this G.P. $a = 10$, $r = 1.50$

$$S_{20} = \frac{a(r^{20}-1)}{(r-1)} = \frac{10(1.50^{20}-1)}{1.50-1}$$

$$= \frac{10(1.50^{20}-1)}{0.50}$$

$$= 66485.1346008$$

(72)

	For AP	For GP
t_n	$a+(n-1)d$	$a \times (r)^{n-1}$
S_n	$\frac{n}{2}[2a+(n-1)d]$ OR $\frac{n}{2}(t_1+t_n)$	$\frac{a(r^n-1)}{(r-1)}$ when $r>1$ $\frac{a(1-r^n)}{(1-r)}$ when $r<1$

(73) $0.60, 0.80, 1.00, 1.20, 1.40 \cdots$ Find t_n, S_{30}

$\Rightarrow$ In this AP $a = 0.60$, $d = 0.20$

① $t_n = a + (n-1)d$

$= 0.60 + (n-1)0.20 = 0.60 + 0.20n - 0.20$

$= 0.40 + 0.20n$

② $S_{30} = \frac{30}{2}\left[1.20 + (29 \times 0.20)\right]$

$= 105$

74 $2 + 6 + 18 + 54 + \text{--------}$ Find S_{20}, t_{28}

$\Longrightarrow$ In this G.P. $a = 2$, $r = 3$

$S_{20} = \frac{a(r^{20}-1)}{(r-1)} = \frac{2(3^{20}-1)}{(3-1)} = 3486784400$

$t_{28} = a \times (r)^{27} = 2 \times 3^{27}$

$= 2 \times 3^3 \times 3^{24}$

$= 2 \times 27 \times 282429536481$

$= 54 \times 282429536481$

75 $1.20 + 1.20^2 + 1.20^3 + 1.20^4 + \text{------}$ Find S_{40}

$\Longrightarrow$ In this GP $a = 1.20$, $r = 1.20$

$S_{40} = \frac{a(r^{40}-1)}{(r-1)} = \frac{1.20(1.20^{40}-1)}{0.20}$

$= 8812.62940725$

76 $100 + 80 + 64 + 51.20 + \text{------}$ Find S_{12}

$\Longrightarrow$ In this G.P. $a = 100$ $r = 0.80$

As $r < 1$ $S_n = \frac{a(1-r^n)}{(1-r)}$

$S_{12} = \frac{100(1-0.80^{12})}{(1-0.80)} = \frac{100(1-0.068719476736)}{0.20}$

$S_{12} = 465.640261635$

(77) $200 + 140 + 98 + 68.60 + \text{-----}$ Find S_{18}

$\Longrightarrow$ In this G.P. $a = 200$, $r = 0.70$

$$S_{18} = \frac{a(1-r^{18})}{(1-r)} = \frac{200(1-0.70^{18})}{1-0.70}$$

$$= \frac{200(1-0.0016284135 9)}{0.30} = 665.581$$

(78) $4 + 6 + 9 + 13.50 + \text{------}$ Find S_{20}, t_{25}

$\Longrightarrow$ In this G.P. $a = 4$, $r = 1.50$

$$S_{20} = \frac{a(r^{20}-1)}{(r-1)} = \frac{4(1.50^{20}-1)}{0.50} = 26594.0538$$

$$t_{25} = a \cdot (r)^{24} = 4 \times (1.50)^{24} = 67,336.45$$

(79) $5 + 10 + 20 + 40 + \text{----------}$ Find sum of infinite terms of this G.P. (i.e. S_∞)

$\Longrightarrow$ In this G.P. $a = 5$, $r = 2$

$$S_\infty = \frac{a(r^\infty - 1)}{(r-1)} = \frac{5(2^\infty - 1)}{(2-1)} = \infty$$

(80) $80 + 40 + 20 + 10 + 5 + \text{------}$ Find S_∞

$\Longrightarrow$ In this G.P. $a = 80$, $r = 0.50$

$$S_\infty = \frac{a(1-r^\infty)}{(1-r)} = \frac{80(1-0.50^\infty)}{1-0.50} = \frac{80(1-0)}{0.50}$$

$$= \frac{80 \times 1}{0.50} = 160$$

Sum of Infinite terms of GP

when $r > 1$

$S_\infty = \infty$

when $0 < r < 1$

$$S_\infty = \frac{a(1-r^{\infty})}{1-r}$$

$$= \frac{a(1-0)}{(1-r)}$$

$$= \frac{a}{(1-r)}$$

(81) $100 + 80 + 64 + 51.20 + \text{--------}$ Find S_∞

$\Longrightarrow$ In this GP $a = 100$, $r = 0.80$

$$S_\infty = \frac{a}{(1-r)} = \frac{100}{1-0.80} = \frac{100}{0.20} = 500$$

(82) $500 + 200 + 80 + 32 + \text{-----}$ Find sum of infinite terms.

$\Longrightarrow$ In this G.P. $a = 500$, $r = 0.40$

$$S_\infty = a/(1-r)$$

$$= \frac{500}{(1-0.40)} = 833.333333 = 833\tfrac{1}{3}$$

Sum of Infinite terms of GP. (S_∞)

when $0 < r < 1$ — $S_\infty = a/(1-r)$

when $r > 1$ — $S_\infty = \infty$

(83) If $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p$ are in G.P.

with common ratio $= r$

then

a, c, e, g, i, k, m, o are also in G.P.

with common ratio $= r^2$

&

a, d, g, j, m, p are also in G.P.

with common ratio $= r^3$

$$\boxed{\begin{array}{l} \text{If } a, b, c, d, e \text{ are in G.P. then} \\ \quad \text{(a) } b, c, d \text{ are 3 G.means bet}^n\ a \ \& \ e \\ \quad \text{(b) } c \text{ is GM of } b, d \\ \quad \text{(c) } c \text{ is GM of } a, e \end{array}}$$

(84) Insert 3 Geometric means between 3 and 7203

$\Longrightarrow$ $3, \bigcirc, \bigcirc, \bigcirc, 7203$

In this GP of 5 terms

$a = 3$, $t_5 = 7203$

$a \cdot r^4 = 7203$

$3 \times r^4 = 7203$

$r^4 = 2401$

$r = (2401)^{1/4} = 7$

$3, 21, 147, 1029, 7203$ are in G.P.

$\therefore$ $21, 147, 1029$ are 3 G.means bet^n 3 & 7203

(85) Insert 5 G.means between 10 & 40960

$\Longrightarrow$

$10, \bigcirc, \bigcirc, \bigcirc, \bigcirc, \bigcirc, 40960$

$a = 10, \quad t_7 = 40960$

$a \cdot r^6 = 40960$

$10 \times r^6 = 40960$

$r^6 = 4096$

$r^6 = 4^6$

$\boxed{r = 4}$

40, 160, 640, 2560, 10240 are 5 G.means between 10 & 40960

(86) Insert 3 G.means between $\frac{1}{9}$ & 9

$\Longrightarrow$

$a = \frac{1}{9}$

$t_5 = 9$

$a \cdot r^4 = 9$

$\frac{1}{9} r^4 = 9$

$r^4 = 81$

$r^4 = 3^4$

$\boxed{r = 3}$

$\frac{1}{9}, \frac{1}{3}, 1, 3, 9$

$\therefore$ $\frac{1}{3}, 1, 3$ are 3 Geometric means between $\frac{1}{9}$ & 9.

(87)

$t_{81} = t_{80} \times r$
$= t_{82} \div r$

$\uparrow$

For GP

$\downarrow$

$t_n \times r = t_{n+1}$

$t_{n+3} \div r = t_{n+2}$

$t_{100} \times r = t_{101}$

$t_{200} \times r^3 = t_{203}$

$t_{500} \div r^4 = t_{496}$

$t_{81} = t_{80} + d$
$= t_{82} - d$

$\uparrow$

For AP

$\downarrow$

$t_n + d = t_{n+1}$

$t_{n+5} - d = t_{n+4}$

$t_{33} + 2d = t_{35}$

$t_{100} - 3d = t_{97}$

(88) 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22 are in A.P.

$\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}, \frac{1}{14}, \frac{1}{16}, \frac{1}{18}, \frac{1}{20}, \frac{1}{22}$ are in H.P.

H.P. $\Rightarrow$ Harmonic progression

If $\frac{p}{4}, \frac{r}{5}, \frac{m}{n}, \frac{18}{23}, \frac{x}{y}$ are in A.P. then

$\frac{4}{p}, \frac{5}{r}, \frac{n}{m}, \frac{23}{18}, \frac{y}{x}$ must be in H.P.

If $\frac{x}{m}, \frac{n}{k}, \frac{l}{r}$ are in H.P. then

$\frac{m}{x}, \frac{k}{n}, \frac{r}{l}$ must be in A.P.

If few terms are in A.P. then their reciprocals must be in H.P. and vice versa

89) If a, b, c are in

A.P. then	G.P. then	H.P. then
		b is HM of a & c
$b =$ AM of a & c	b is GM of a & c	$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ must be in AP
	$\frac{b}{a} = \frac{c}{b}$	
$b - a = c - b$		$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$
$b + b = a + c$	$b^2 = ac$	
$2b = a + c$	$b = \sqrt{ac}$	$\frac{1}{b} + \frac{1}{b} = \frac{1}{c} + \frac{1}{a}$
$b = \frac{a+c}{2}$	GM of a & c $= \sqrt{ac}$	$\frac{2}{b} = \frac{a+c}{ac}$
AM of a & c $= \left(\frac{a+c}{2}\right)$	GM of x & y $= \sqrt{xy}$	$\frac{b}{2} = \frac{ac}{a+c}$
AM of x & y $= \left(\frac{x+y}{2}\right)$	GM of p, q $= \sqrt{pq}$	$b = \left(\frac{2ac}{a+c}\right)$
	GM of d, e $= \sqrt{de}$	
AM of p, q $= \left(\frac{p+q}{2}\right)$		HM of a & c $= \frac{2ac}{a+c}$
		HM of x, y $= \frac{2xy}{x+y}$
		HM of p, q $= \left(\frac{2pq}{p+q}\right)$

90) Find AM, GM, HM of 60, 30

$\Longrightarrow$

$$AM = \frac{60+30}{2} = 45$$

$$GM = \sqrt{60 \times 30} = \sqrt{1800} = 42.42641$$

$$HM = \left(\frac{2 \times 60 \times 30}{60+30}\right) = \left(\frac{3600}{90}\right) = 40$$

(91) Mr. A travels pune to Mumbai a distance of 200 kms at a uniform speed of 100 kms/hr & return at uniform speed of 50 kms/hr Find avg. speed of whole journey?

$$\Rightarrow \text{Avg. speed} = \left(\frac{\text{Total distance traveled}}{\text{Total time taken}}\right)$$

$$= \frac{200\text{ kms} + 200\text{ kms}}{\left(\frac{200\text{ kms}}{100\text{ kms/hr}}\right) + \left(\frac{200\text{ kms}}{50\text{ kms/hr}}\right)}$$

$$= \frac{400\text{ kms}}{2\text{ hrs} + 4\text{ hrs}} = \frac{400\text{ kms}}{6\text{ hrs}}$$

$$= 66.666666\text{ kms/hr}$$

(92) Find HM of 100, 50

$$\Rightarrow \text{HM} = \frac{2 \times 100 \times 50}{100 + 50}$$

$$= \frac{10{,}000}{150} = 66.666666$$

HM is useful in calculation of avg speed of a journey

(93) If S_n for AP is $(20n^2 - 13n)$ Find t_n

$\Longrightarrow$ $S_n = 20n^2 - 13n$

$S_1 = 20(1)^2 - 13(1) = 7$ $\quad a = 7$

$S_2 = 20(2)^2 - 13(2) = 54$ $\quad t_2 = 47$

$d = 40$

$t_n = a + (n-1)d$

$= 7 + (n-1)40$

$= 7 + 40n - 40$

$= 40n - 33$

(94) For AP If $90 \times t_{90} = 100 \times t_{100}$

Find t_{190}

$\Longrightarrow$ zero

3 Imp. short cuts of A.P.

If $m \times t_m = n \times t_n$ then $t_{m+n} = \text{zero}$
If $t_m = n$ & $t_n = m$ then $t_{m+n} = \text{zero}$
If $S_m = S_n$ then $S_{m+n} = 0$

(95) $(243)^{1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots\cdots \infty \text{ terms}} = ?$

$\Longrightarrow$ $(243)^{1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots \infty \text{ terms}}$

$= (243)^{a/1-r}$ $\quad a = 1$, $r = 1/3$

$= (243)^{\frac{1}{1-\frac{1}{3}}} = (3^5)^{\frac{1}{2/3}} = (3^5)^{3/2} = \left(3^{15/2}\right)$

(96) If $x = 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots\cdots$

$y = \frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \frac{1}{625} + \cdots\cdots$

Find value of (xy)

$\Longrightarrow$ $x = 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots\cdots$

$$x = \frac{1}{1-\frac{1}{4}} = \frac{1}{3/4} = 4/3$$

$y = \frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \cdots\cdots$

$$y = \frac{\frac{1}{5}}{1-\frac{1}{5}} = \frac{\frac{1}{5}}{\frac{4}{5}} = \frac{1}{4}$$

$\therefore\ xy = \frac{4}{3} \times \frac{1}{4} = 1/3$

(97) Find AM, GM, HM of p, q

$AM = \left(\frac{p+q}{2}\right)$ $\quad$ $GM = \sqrt{pq}$ $\quad$ $HM = \left(\frac{2pq}{p+q}\right)$

(98) For 2 observations (positive), $GM^2 = AM \times HM$

----- True/False

$\Longrightarrow$ Let 2 obs^n^s be x & y

L.H.S. $= GM^2 = (\sqrt{xy})^2 = xy$

R.H.S. $= AM \times HM = \frac{\cancel{(x+y)}}{\cancel{2}} \times \frac{\cancel{2}xy}{\cancel{(x+y)}} = xy$

AS L.H.S. = R.H.S.

$\therefore\ GM^2 = AM \times HM$

(99) For 2 observations GM = 9, AM = 10, HM = ?

$\Longrightarrow$

$$GM^2 = AM \times HM$$
$$9^2 = 10 \times HM$$
$$HM = 81/10 = 8.10$$

(100) (i) If $b^2 = ac$ $\downarrow$ b is GM of a & c

(ii) If $GM^2 = AM \times HM$ $\downarrow$ GM is GM of AM & HM

(101) $3 + 33 + 333 + 3333 + 33333 + \ldots\ldots$ n terms = ?

$= 3\,(1 + 11 + 111 + 111 + \ldots\ldots\ldots \text{ n terms})$

$= \frac{3}{9} \times 9 \times (1 + 11 + 111 + 1111 + \ldots\ldots \text{ n terms})$

$= \frac{3}{9} \times (9 + 99 + 999 + 9999 + \ldots\ldots \text{ n terms})$

$= \frac{3}{9}\left[(10-1) + (100-1) + (1000-1) + (10{,}000-1) + \ldots\ldots \text{ n terms}\right]$

$= \frac{3}{9}\left[(10 + 100 + 1000 + \ldots\ldots \text{ n terms}) - (1 + 1 + 1 + 1 + \ldots \text{ n terms})\right]$

$= \frac{3}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$

QUESTIONS FOR PRACTICE

① $9x+13,\ 15x-33,\ 18x+88$ are in A.P.
Find value of x

$\Longrightarrow$ $15x-33-9x-13 = 18x+88-15x+33$

$6x-46 = 3x+121$

$3x = 167$

$x = \left(\frac{167}{3}\right) = 55.666666$

$x = 55\frac{2}{3}$

② Insert 5 A.means between (-36) and 1296.

$\Longrightarrow$ $a=-36$ $\quad t_7 = 1296$

$a+6d = 1296$

$-36+6d = 1296$

$d = 222$

$\therefore$ 5 A.Means are : 186, 408, 630, 852, 1074

③ For 2 observations $GM=9$, $AM=12$, $HM=?$

$\Longrightarrow$ $GM^2 = AM \times HM$

$9^2 = 12 \times HM$

$HM = 81/12 = 27/4 = 6.75$

④ $55^2+56^2+57^2+ \text{-------} + 130^2 = ?$

$\Longrightarrow$ $= (1^2+2^2+3^2+ \text{-----} +130^2) - (1^2+2^2+ \text{----} +54^2)$

$= \left(\frac{130 \times 131 \times 261}{6}\right) - \left(\frac{54 \times 55 \times 109}{6}\right)$

$= 6,86,850/-$

⑤ sum of all terms of AP = 7171

$t_1 = -4$ & $t_n = 146$

Find No. of terms & common difference.

$\Longrightarrow$ $S_n = \frac{n}{2}(t_1 + t_n)$

$7171 = \frac{n}{2}(-4 + 146)$

$7171 = n \times 71$

$n = 101$

$t_{101} = 146$

$a + 100d = 146$

$-4 + 100d = 146$

$100d = 150$

$d = 1.50 = 3/2$

$\therefore$ No. of terms in AP = 101

common difference = 1.50

⑥ If $a^{1/x} = b^{1/y} = c^{1/z}$ and a, b, c are in G.P. then x, y, z are in :

~~(a)~~ AP (b) GP (c) Both (d) None

$\Longrightarrow$ $a^{1/x} = b^{1/y} = c^{1/z} = k$

$a^{1/x} = k$

$(a^{1/x})^x = k^x$

$\boxed{a = k^x}$

$\therefore b = k^y$

$c = k^z$

a, b, c are in G.P.

$b^2 = ac$

$(k^y)^2 = k^x . k^z$

$k^{2y} = k^{x+z}$

$\therefore 2y = x + z$

$y + y = x + z$

$y - x = z - y$

$\therefore$ x, y, z are in A.P.

⑦ If G is GM of a & b. Find the value of $\left[\frac{1}{G^2 - a^2} + \frac{1}{G^2 - b^2}\right]$

(a) G^2 (b) $3G^2$ ~~(c)~~ $1/G^2$ (d) $2/G^2$

$\Longrightarrow$ $G^2 = ab$ --- ①

$\frac{1}{ab - a^2} + \frac{1}{ab - b^2} = \frac{1}{a(b-a)} + \frac{1}{-b(b-a)}$

$= \frac{1 \times b}{ab(b-a)} - \frac{1 \times a}{ba(b-a)} = \frac{\cancel{(b-a)}}{ab\cancel{(b-a)}} = \left(\frac{1}{G^2}\right)$

(8) $243 \times 243^{1/6} \times 243^{1/36} \times \ldots\ldots \infty$ terms = ?

(a) 3 (b) ∞ (c) 243 ~~(d)~~ 729

$= 243^{1} \times 243^{1/6} \times 243^{1/36} \times \ldots\ldots \infty$ terms

$= (243)^{\left(1+\frac{1}{6}+\frac{1}{36}+\ldots\ldots \infty \text{ terms}\right)}$

$= (243)^{\frac{1}{1-1/6}} = (243)^{\frac{1}{5/6}} = (3^5)^{6/5} = 3^6 = 729$

(9) If a, b, c are in A.P. and x, y, z are in G.P.

then $(x)^{b-c} \cdot (y)^{c-a} \cdot (z)^{a-b} = ?$

~~(a)~~ 1 (b) 0 (c) 2 (d) None of these

$\Longrightarrow$ $a = 2$ $x = 3$

$b = 4$ $y = 9$

$c = 6$ $z = 27$

$(x)^{b-c} \cdot (y)^{c-a} \cdot (z)^{a-b}$

$= (3)^{-2} \times (9)^{4} \times (27)^{-2} = 3^{-2} \times 3^{8} \times 3^{-6} = (3)^{-2+8-6} = 3^{0}$

$= 1$

(10) For G.P. $t_5 = (3)^{1/3}$. Find product of first 9 terms of G.P.

$\Longrightarrow$ $t_5 = a \cdot r^4 = (3)^{1/3}$ ---- (1)

product of first 9 terms of G.P.

$= a \times ar \times ar^2 \times ar^3 \times \ldots\ldots \times ar^8$

$= a^9 \times r^{1+2+3+4+5+6+7+8}$

$= a^9 \times (r)^{\frac{8 \times 9}{2}}$

$= a^9 \cdot r^{36}$

$= (a \cdot r^4)^9$

$= (3^{1/3})^9$

$= 3^3$

$= 27$

(11) Sum of 3rd & 9th term of AP is 8. Find S_{11}.

~~(a)~~ 44 (b) 22 (c) 19 (d) 11

$\Longrightarrow$ $t_3 + t_9 = 8$

$a + 2d + a + 8d = 8$

$2a + 10d = 8$ ---- (1)

$$S_{11} = \frac{11}{2}\left[2a + 10d\right] = \frac{11}{2} \times 8 = 44$$

(12) $1 + 3 + 5 + 7 + 9 + \cdots\cdots + 513 = ?$

$\Longrightarrow$ $a = 1, \ d = 2, \ t_n = 513$

$1 + (n-1)2 = 513$

$n = 257$

$= 1 + 3 + 5 + 7 + \cdots + 513 =$ sum of first 257 odd numbers

$= 257^2$

$= 66049$

(OR) $S_{257} = \frac{257}{2}(1 + 513) = 66049$

(13) $10 + 9\frac{2}{3} + 9\frac{1}{3} + 9 + \cdots\cdots$ n terms $= 155$ then $n = ?$

(a) 30 (b) 31 ~~(c)~~ a or b (d) None of these

$\Longrightarrow$ $\frac{30}{3}, \frac{29}{3}, \frac{28}{3}, \frac{27}{3}, \cdots\cdots$

In this A.P. $a = 10, \ d = -1/3$

$$S_n = 155 = \frac{n}{2}\left[20 + (n-1)\left(-\frac{1}{3}\right)\right]$$

$$155 = \frac{n}{2}\left[\frac{60}{3} - \frac{n}{3} + \frac{1}{3}\right]$$

$$155 = \frac{n}{2}\left(\frac{60-n+1}{3}\right)$$

$$155 = \frac{n}{2}\left(\frac{61-n}{3}\right)$$

$$930 = n(61-n)$$

$$930 = 61n - n^2$$

$$n^2 - 61n + 930 = 0$$

$$(n-30)(n-31) = 0 \quad \therefore n = 30,\ n = 31$$

(14) Insert 5 Geometric means between 10 & 7290.

$\Longrightarrow$ $a = 10,\ t_7 = 7290$

$$a \cdot r^6 = 7290$$

$$10 \times r^6 = 7290$$

$$r^6 = 729 = 3^6$$

$$\therefore r = 3$$

$\therefore$ 5 G.Means are : 30, 90, 270, 810, 2430

(15) If a, b, c, d, e, f, g, h are in A.P. with common difference = 10 then common diff for A P b, d, f, h is

(a) 10 (b) 10^2 (c) 20 (d) None of these

(16) If a, b, c, d, e, f, g, h are in G.P. with common ratio = 4, then common ratio for G.P. a, c, e, g is

(a) 4 (b) 4^2 (c) 8 (d) None of these

(17) 2 A. Means between -6 & 14 are

~~(a)~~ $2/3, 22/3$ (b) $2/3, 1/3$ (c) 0, 8 (d) None of these

$\Longrightarrow$ $a = -6$, $t_4 = 14$

$a + 3d = 14$

$-6 + 3d = 14$

$3d = 20$

$d = 6.6666666$

$\therefore$ 2 A.Means are : $0.6666666666, 7.33333333$

: $2/3, 22/3$

(18) Find sum of all 5 digit even natural numbers divisible by 7

$\Longrightarrow$ $10{,}010 + 10{,}024 + 10{,}038 + \text{---------} + 99988 = ?$

$99988 = 10{,}010 + (n-1)14$

$n = 6428$

$$S_{6428} = \frac{6428}{2}(10{,}010 + 99988)$$

$= 35{,}35{,}33{,}572$

(19) $-8, -6, -4, -2$ ----- in this AP $S_n = 52$

Find 'n'

(a) 14 ~~(b)~~ 13 (c) 4 (d) None of these

$\Longrightarrow$ In this A.P. $a = -8$, $d = 2$

$$S_n = 52 = \frac{n}{2}\left[-16 + (n-1)2\right]$$

$104 = n(-16 + 2n - 2)$

$104 = n(2n - 18)$

$$104 = n \times 2 \times (n-9)$$
$$52 = n(n-9)$$
$$\therefore \quad n = 13$$

(20) For G.P. $t_3 = 12$ & $t_5 = 48$ Find t_2

(a) 2 (b) 36 ~~(c)~~ 6 (d) None

$\Longrightarrow$

$$t_3 \times r \times r = t_5$$
$$a \cdot r^2 \times r \times r = 48$$
$$12 \times r^2 = 48$$
$$r^2 = 4$$
$$\boxed{r = 2}$$

$$\frac{t_3}{r} = t_2$$
$$\frac{12}{2} = t_2$$
$$\therefore t_2 = 6$$

(21) 1, x, 9 are in G.P. then $x = ?$

(a) 3 (b) −3 ~~(c)~~ (a) OR (b) (d) None

1, 3, 9 are in G.P.

1, −3, 9 are also in G.P.

(22) A man saved ₹ 16,500 in 10 years. In each year he saved ₹ 100 more than he saved in preceding year. How much did he save in the first year?

~~(a)~~ ₹ 1200 (b) ₹ 1300 (c) ₹ 1550 (d) None of these

$\Longrightarrow$ First year saving $= a$

$a + (a+100) + (a+200) + (a+300) + \text{-----}$ 10 terms $= 16500$

$$S_{10} = \frac{10}{2}\left[2a + (9 \times 100)\right] = 16500$$
$$2a + 900 = 3300$$
$$2a = 2400$$
$$a = 1200$$

(23) 120 + 60 + 30 + 15 + ------- Find S_∞

$\Longrightarrow$ In this G.P. $a = 120$, $r = 0.50$

$$S_\infty = \left[\frac{a}{(1-r)}\right] = \frac{120}{1-0.50} = \left(120/0.50\right) = 240$$

(24) 60 + 6 + 0.60 + 0.06 + ------ Find sum of infinite terms.

$\Longrightarrow$ In this G.P. $a = 60$, $r = 0.10$

$$S_\infty = \left[\frac{a}{1-r}\right] = \left(\frac{60}{1-0.10}\right) = 66.6666666666$$
$$= 66\tfrac{2}{3}$$

(25) For GP $t_7 = 40,820$ and $t_9 = 63,781.25$ Find common ratio.

~~(a)~~ 1.25 (b) −1.30 (c) 1.50 (d) None

$\Longrightarrow$ $t_7 \times r \times r = t_9$

$$40820 \times r^2 = 63781.25$$
$$r^2 = 1.5625$$
$$r = 1.25$$

(26) $(5+x)$, $(7+x)$, $(18+x)$ are in A.P. Find x

(a) 2 (b) 0 (c) 10 ~~(d)~~ wrong data in question

$\Longrightarrow$ $(5+x), (7+x), (18-x)$

$$t_2 - t_1 = 7 + \cancel{x} - 5 - \cancel{x} = 2$$
$$t_3 - t_2 = 18 + \cancel{x} - 7 - \cancel{x} = 11$$

$\therefore$ $(5+x), (7+x), (18+x)$ are in A.P. $\Longrightarrow$ This statement is wrong

(27) For AP, (First term = common diff) then what is the ratio of m^{th} term to n^{th} term

~~(a)~~ $m:n$ (b) $n:m$ (c) $m^2 : n^2$ (d) None

$\Rightarrow$ For AP $\boxed{a=d}$

$$\frac{t_m}{t_n} = \frac{a+(m-1)d}{a+(n-1)d} = \frac{a+md-d}{a+nd-d} = \frac{\cancel{d}+md-\cancel{d}}{\cancel{d}+nd-\cancel{d}}$$

$$= \frac{md}{nd} = \frac{m}{n} = m:n$$

(28) For AP sum of first 50 terms is equal to sum of first 60 terms then sum of first 110 terms is :

(a) 110 ~~(b)~~ zero (c) 250 (d) None

(29) n^{th} term of the sequence 16, 8, 4, 2 is $\left(1/2^{17}\right)$. Find value of 'n'.

(a) 20 (b) 21 ~~(c)~~ 22 (d) None

$\Rightarrow$ 16, 8, 4, 2 ---- In this G.P. $a=16$, $r=1/2$

$$t_n = \frac{1}{2^{17}}$$

$$a \times (r)^{n-1} = \frac{1}{2^{17}}$$

$$16 \times \left(\frac{1}{2}\right)^{n-1} = \frac{1}{2^{17}}$$

$$2^4 \times \frac{1}{2^{n-1}} = \frac{1}{2^{17}}$$

$$\left(\frac{1}{2^{n-1}}\right) = \frac{1}{2^{17}} \times \frac{1}{2^4} = \left(\frac{1}{2^{21}}\right)$$

$\therefore n-1 = 21 \quad \therefore n = 21+1$

$n = 22$

(30) In AP terms increase or decrease by a Fixed number. This statement is :

~~(a)~~ True (b) False

(31) For AP $t_9 = -6$ and common difference $= 5/4$

Find t_{25}

(a) 12 ~~(b)~~ 14 (c) 16 (d) 18

$\Longrightarrow$

$a + 8d = -6$

$a + \left(8 \times \frac{5}{4}\right) = -6$

$a + 10 = -6$

$a = -16$

t_{25}

$= a + 24d$

$= -16 + \left(24 \times 5/4\right)$

$= -16 + 30$

$= 14$

(32) Ratio of 7^{th} to 3^{rd} term of AP is 12 : 5 then Find ratio of 13^{th} to 4^{th} term ?

(a) 8:5 (b) 9:4 (c) 7:3 ~~(d)~~ 10:3

$\Longrightarrow$

$\frac{t_7}{t_3} = \frac{a+6d}{a+2d} = \frac{12}{5}$

$5a + 30d = 12a + 24d$

$\boxed{6d = 7a}$

$\frac{t_{13}}{t_4} = \frac{a+12d}{a+3d}$

$= \frac{a + 2(6d)}{a + 0.50(6d)}$

$= \frac{a + (2 \times 7a)}{a + (0.50 \times 7a)} = \frac{a + 14a}{a + 3.50a}$

$= \frac{15a}{4.50a} = \frac{15}{4.50} = \frac{150}{45}$

$= 10 : 3$

(33) Sum of a series in AP is 525. 1^{st} term is 3 and last term is 39. Find common difference.

~~(a)~~ 3/2 (b) 2/3 (c) 1/3 (d) None of these

$\Longrightarrow$

$a = t_1 = 3$

$t_n = 39$

$S_n = 525$

$S_n = \frac{n}{2}(t_1 + t_n)$

$525 = \frac{n}{2}(3 + 39)$

$525 = n \times 21$

$n = 25$

$t_{25} = 39 = a + 24d = 3 + 24d \quad \therefore 24d = 36$

$d = \frac{36}{24} = \frac{3}{2}$

(34) Find common difference of AP whose first term is 100 and sum of whose first 6 terms is 5 times the sum of next 6 terms?

~~(a)~~ −10 (b) −15 (c) 10 (d) None of these

$\Longrightarrow \quad S_6 = 5 \times (S_{12} - S_6)$

$\frac{3(200+5d)}{5} = 6(200+11d) - 3(200+5d)$

$120 + 3d = 1200 + 66d - 600 - 15d$

$120 + 3d = 51d + 600$

$-480 = 48d$

$\therefore \; d = -10$

(35) which term of G.P. 5, 10, 20, 40 ------ is 1280.

(a) 11th ~~(b)~~ 9th (c) 8th (d) 12th

$a = 5, r = 2$

$t_n = a(r)^{n-1} = 1280$

$5 \times (2)^{n-1} = 1280$

$\therefore \; 2^{n-1} = 256 = 2^8$

$\therefore \; n-1 = 8 \quad \therefore n = 9$

5, 10, 20, 40, 80, 160, 320, 640, 1280

(36) Find AM, GM, HM of 20 & 50

$AM = \left(\frac{20+50}{2}\right)$ $\quad GM = \sqrt{20 \times 50}$ $\quad HM = \frac{2 \times 20 \times 50}{20+50}$

$= 35$ $\quad GM = 31.6227766016$ $\quad HM = 28.571429$

(37) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}$ are in

(a) A.P. (b) G.P. ~~(c)~~ H.P. (d) M.P.

$\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}$ are in H.P. as

their reciprocals 2, 4, 6, 8, 10 are in A.P.

(38) If x, y, z are in H.P. then $1/x, 1/y, 1/z$ are in :

~~(a)~~ A.P. (b) G.P. (c) H.P. (d) None

(39) If a, b, c are in G.P. then a^2, b^2, c^2 are in

(a) A.P. ~~(b)~~ G.P. (c) H.P. (d) None

2, 4, 8 are in G.P. $\therefore$ 4, 16, 64 are also in G.P.

5, 20, 80 are in G.P. $\therefore$ 25, 400, 6400 are also in G.P.

(40) If one AM 'A' and 2 Geometric means G_1 & G_2 are inserted between any 2 numbers then $(G_1^3 + G_2^3) = ?$

(a) $2G_1 \cdot G_2$ ~~(b)~~ $2A \cdot G_1 \cdot G_2$ (c) $2G_1/A$ (d) None

$\Longrightarrow$ Let those 2 numbers be x & y

$\therefore$ x, A, y are in A.P.

$\therefore A = \dfrac{x+y}{2}$

$\therefore x + y = 2A$

x, G_1, G_2, y are in G.P.

$\left(\dfrac{G_1}{x}\right) = \left(\dfrac{G_2}{G_1}\right) = \left(\dfrac{y}{G_2}\right)$

$\therefore G_1^2 = x \cdot G_2$ & $G_2^2 = y \cdot G_1$

$$G_1^3 + G_2^3 = G_1 \cdot G_1^2 + G_2 \cdot G_2^2$$
$$= G_1 \cdot x \cdot G_2 + G_2 \cdot y \cdot G_1$$
$$= G_1 \cdot G_2 (x+y)$$
$$= G_1 \cdot G_2 \cdot 2A = 2AG_1G_2$$

(41) AM of 2 numbers is 15 and their GM is 9. then the numbers are :

~~(a)~~ 27, 3 (b) 9, 9 (c) 16, 9 (d) None

$AM = \dfrac{27+3}{2} = 15$

$GM = \sqrt{27 \times 3} = 9$

(42) If a, b, c are in G.P., a, x, b and b, y, c are both in A.P. then $\left(\frac{a}{x} + \frac{c}{y}\right) = ?$

(a) 1 (b) 0 ~~(c)~~ 2 (d) None of these

$\Rightarrow$

$a = 2$	$a = 2$	$b = 4$
$b = 4$	$x = 3$	$y = 6$
$c = 8$	$b = 4$	$c = 8$

$$\frac{a}{x} + \frac{c}{y}$$

$$= \frac{2}{3} + \frac{8}{6}$$

$$= \frac{4}{6} + \frac{8}{6} = \frac{12}{6} = 2$$

(43) Find t_{25} for AP If $t_9 = -6$ and common difference is $9/4$

$\Rightarrow$

$$t_9 = a + 8d = -6$$

$$a + (8 \times 9/4) = -6$$

$$a + 18 = -6$$

$$a = -24$$

$$t_{25} = a + 24d$$

$$= -24 + (24 \times 9/4)$$

$$= 30$$

(44) If $(p+1)^{th}$ term of AP is twice the $(q+1)^{th}$ term then Ratio of $(p+q+1)^{th}$ term to $(3p+1)^{th}$ term is

~~(a)~~ 1:2 (b) 2:1 (c) 1:3 (d) None

$\Rightarrow$

$$t_{p+1} = 2 \times t_{q+1}$$

$$a + pd = 2(a + qd)$$

$$a + pd = 2a + 2qd$$

$$pd - 2qd = a$$

$$\frac{t_{p+q+1}}{t_{3p+1}} = \frac{a + (p+q)d}{a + 3pd}$$

$$= \left(\frac{pd - 2qd + pd + qd}{pd - 2qd + 3pd}\right)$$

$$= \left(\frac{2pd - qd}{4pd - 2qd}\right) = \frac{\cancel{(2pd - qd)}}{2\cancel{(2pd - qd)}}$$

$$= \frac{1}{2} = 1:2$$

(45) 4^{th} term of AP is equal to 3 times of first term and 7^{th} term exceeds twice of third term by 1. Find First term.

~~(a)~~ 3 (b) 5 (c) 7 (d) 9

$\Longrightarrow$ $a+3d = 3a$

$-2a+3d = 0$ ---- (1)

$a+6d = 2(a+2d)+1$

$a+6d = 2a+4d+1$

$-a+2d = 1$ ---- (2)

$-4a+6d = 0$

$\mp 3a \pm 6d = \mp 3$

$-a = -3$

$a = 3$

(46) In G.P. $S_n = 4095$, $r=2$, $t_n = 2048$ Find 'n'.

(a) 10 (b) 11 ~~(c)~~ 12 (d) 15

$\Longrightarrow$

$t_n = a(2)^{n-1} = 2048$

$a \times \frac{2^n}{2^1} = 2048$

$a \times 2^n = 4096$

$4095 = \frac{a(2^n-1)}{2-1}$

$4095 = a \times 2^n - a$

$4095 = 4096 - a$

$a = 1$

$a \times 2^n = 4096$

$1 \times 2^n = 4096$

$2^n = 2^{12}$

$\therefore n = 12$

(47) Find 6^{th} term from end for G.P.

$8, 4, 2, 1, \ldots\ldots, 1/1024$

(a) 1/64 ~~(b)~~ 1/32 (c) 32 (d) None of these

$\Longrightarrow$

$8, 4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \frac{1}{256}, \frac{1}{512}, \frac{1}{1024}$

(48) For G.P. $t_4 = x$, $t_{10} = y$, $t_{16} = z$ then

(a) $x^2 = y.z$ (b) $y^2 = x.z$ (c) $z^2 = x.y$ (d) $xyz = 1$

$\Longrightarrow$ $t_4 = a.r^3 = x$
$t_{10} = a.r^9 = y$
$t_{16} = a.r^{15} = z$

$y^2 = (a.r^9)^2 = (a^2.r^{18})$
$x.z = a.r^3 \times a.r^{15} = (a^2.r^{18})$
$\therefore y^2 = xz$

(49) 2 + 5 + 10 + 17 + 26 + ------- Find sum of 100 terms of this series.

$\Longrightarrow$ $2 + 5 + 10 + 17 + 26 + \cdots$ 100 terms

$= (1^2+1) + (2^2+1) + (3^2+1) + (4^2+1) + (5^2+1) + \cdots + (100^2+1)$

$= (1^2 + 2^2 + 3^2 + \cdots + 100^2) + (1+1+1+1 \cdots \text{100 terms})$

$= \left(\frac{100 \times 101 \times 201}{6}\right) + 100$

$= 3,38,450$

(50) 0 + 6 + 24 + 60 + 120 + ------ Find sum of 80 terms of this series.

$\Longrightarrow = (1^3-1) + (2^3-2) + (3^3-3) + (4^3-4) + (5^3-5) + \cdots + (80^3-80)$

$= (1^3 + 2^3 + 3^3 + \cdots + 80^3) - (1+2+3+4+ \cdots + 80)$

$= \left(\frac{80 \times 81}{2}\right)^2 - \left(\frac{80 \times 81}{2}\right)$

$= 10497600 - 3240 = 10494360/-$

(51) $3\frac{1}{2}, 7, 10\frac{1}{2}, 14, \cdots$ Find S_{60}

$\Longrightarrow$ $3.50 + 7 + 10.50 + 14 + \cdots$ 60 terms = ?

In this AP $a = 3.50$, $d = 3.50$

$S_{60} = \frac{60}{2}(2a + 59d) = 30(7 + 59 \times 3.50) = 6405$

52) $2 + 12 + 36 + 80 + 150 + \cdots$

Find sum of first 70 terms of this series.

$\Longrightarrow$

$= (1^2+1^3) + (2^2+2^3) + (3^2+3^3) + (4^2+4^3) + (5^2+5^3) \cdots + (70^2+70^3)$

$= (1^2+2^2+3^2+\cdots+70^2) + (1^3+2^3+3^3+\cdots+70^3)$

$= \left(\frac{70 \times 71 \times 141}{6}\right) + \left(\frac{70 \times 71}{2}\right)^2$

$= 62,92,020$

53) For G.P. $a = 729$, $t_7 = 64$ Find S_7

(a) 2259 (b) 3059 (c) 2059 (d) 2459

$\Longrightarrow$ $a = 729$

$t_7 = a \cdot r^6 = 64$

$729 \times r^6 = 64$

$r^6 = \frac{64}{729}$

$r^6 = \left(\frac{2}{3}\right)^6$

$\therefore r = 2/3$

$S_7 = \left[\frac{a(1-r^7)}{1-r}\right]$

$= \left[\frac{729\left(1-\frac{128}{2187}\right)}{1-\frac{2}{3}}\right] = \frac{729 \times \frac{2059}{2187}}{\frac{1}{3}} = 2059$

54) a, b, c are A.P. as well as GP then

(a) $a = b = c$ (b) $a > b < c$ (c) $a = b \neq c$ (d) None

55) sum of first '2n' terms of AP 2, 5, 8 ----- is equal to sum of first 'n' terms of AP 57, 59, 61, ------- then $n = ?$

(a) 10 (b) 12 (c) 11 (d) 13

$\Longrightarrow$ $\frac{\cancel{2}\cancel{n}}{\cancel{2}}\left[4 + (2n-1)3\right] = \frac{\cancel{n}}{2}\left[114 + (n-1)2\right]$

$$4+6n-3 = \frac{114+2n-2}{2}$$

$$2(6n+1) = 112+2n$$

$$12n+2 = 112+2n$$

$$10n = 110$$

$$\boxed{n = 11}$$

(56) $2+1+\frac{1}{2}+\frac{1}{4}+ \text{---------} = ?$

$\Longrightarrow$ In this G.P. $a=2$, $r = 0.50$

$$S_\infty = \frac{a}{(1-r)} = \frac{2}{1-0.50} = \frac{2}{0.50} = 4$$

(57) 4, x, 36 are in G.P. then $x = ?$

~~(a)~~ 12 (b) 24 (c) 20 (d) None

$\Longrightarrow$ 4, x, 36 are in GP

$$\frac{x}{4} = \frac{36}{x} \quad \therefore x^2 = 4\times 36 = 144$$

$$x = \sqrt{144} = \pm 12$$

(58) 3 numbers are in G.P. If we double the middle term, we get A.P. then common ratio of G.P. is equal to :-

~~(a)~~ $2\pm\sqrt{3}$ (b) $3\pm\sqrt{2}$ (c) $3\pm\sqrt{5}$ (d) $5\pm\sqrt{3}$

$\Longrightarrow$ Let $\left[\frac{a}{r}, a, ar\right]$ are 3 terms are in G.P.

$\therefore \frac{a}{r}, 2a, ar$ are in A.P.

$$2a - \frac{a}{r} = ar - 2a$$

$$\cancel{a}\left(2-\frac{1}{r}\right) = \cancel{a}(r-2)$$

$$\therefore 2-\frac{1}{r} = r-2$$

For a quadratic eqn

$$ax^2+bx+c=0$$

$$x = \left(\frac{-b\pm\sqrt{b^2-4ac}}{2a}\right)$$

$2r - 1 = r^2 - 2r$

$0 = r^2 - 2r - 2r + 1$

$\therefore \; r^2 - 4r + 1 = 0$

$r = \frac{4 \pm \sqrt{16 - 4(1)(1)}}{2 \times 1} = \frac{4 \pm \sqrt{12}}{2}$

$r = \frac{4 \pm \sqrt{4 \times 3}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = \frac{2(2 \pm \sqrt{3})}{2}$

$r = 2 \pm \sqrt{3}$

59) $50 + 45 + 40 + \text{-------- } n$ terms = zero

then $n = ?$

$\Longrightarrow$ In this AP $a = 50$, $d = -5$, $S_n = 0$, $n = ?$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$0 = \frac{n}{2}\left[100 + (n-1)(-5)\right]$

$0 = 100 - 5n + 5$

$\therefore \; 105 - 5n = 0$

$105 = 5n$

$\therefore \; n = 21$

$\therefore$ sum of 21 terms of above AP is zero

60) If you save 1 paise today, 2 paise the next day 4 paise the succeeding day and so on, then your total savings in two weeks will be

(a) ₹ 163 (b) ₹ 183 ~~(c)~~ ₹ 163.83 (d) none of these

$\Longrightarrow$ $1 + 2 + 4 + 8 + \text{------ } 14$ terms $= ?$

$S_{14} = \frac{1(2^{14} - 1)}{(2-1)} = 16{,}383$ paise $=$ ₹ 163.83

61) The sum of the first 20 terms of a G. P is 244 times the sum of its first 10 terms. The common ratio is

(a) $\pm\sqrt{3}$ (b) ± 3 ~~(c)~~ $\sqrt{3}$ (d) none of these

$\Longrightarrow$

$S_{20} = 244 \times S_{10}$

$\frac{a(r^{20} - 1)}{\cancel{(r-1)}} = 244 \times \frac{a \times (r^{10} - 1)}{\cancel{(r-1)}}$

$\cancel{a}\left[(r^{10})^2 - 1^2\right] = 244 \times \cancel{a} \times (r^{10} - 1)$

$\cancel{(r^{10} - 1)}(r^{10} + 1) = 244\,\cancel{(r^{10} - 1)}$

$r^{10} + 1 = 244$

$r^{10} = 243$

$r^{10} = 3^5$

$r^{10} = \left[(\sqrt{3})^2\right]^5$

$r^{10} = (\sqrt{3})^{10}$

$\therefore \; r = \sqrt{3}$

62) The product of 3 numbers in G P is 729 and the sum of squares is 819. The numbers are

(a) 9, 3, 27 (b) 27, 3, 9 ~~(c)~~ 3, 9, 27 (d) none of these

3, 9, 27 are in G.P.

$3 \times 9 \times 27 = 729$, $3^2 + 9^2 + 27^2 = 819$

63 & 64

The number of terms to be taken so that 1 + 2 + 4 + 8 + will be 8191 is

(a) 10 ~~(b)~~ 13 (c) 12 (d) none of these

Four geometric means between 4 and 972 are

~~(a)~~ 12, 36, 108, 324 (b) 12, 24, 108, 320 (c) 10, 36, 108, 320 (d) none of these

63 $1 + 2 + 4 + 8 + \dots$ n terms $= 8191$

$S_n = 8191 = \frac{a(r^n - 1)}{(r-1)}$

$8191 = \frac{1(2^n - 1)}{(2-1)}$

$8191 = 2^n - 1$

$\therefore 2^n = 8192$

$2^n = 2^{13}$

$\therefore \boxed{n = 13}$

64 4, , , , , 972

$a = 4$, $t_6 = 972$

$a \cdot r^5 = 972$

$4 \times r^5 = 972$

$r^5 = 243$

$r^5 = 3^5$

$r = 3$

$\therefore$ 4 G.means are :

12, 36, 108, 324

65 $4 + 0.40 + 0.04 + 0.004 + \text{-----} \infty$ terms = ?

$\Longrightarrow$ In this G.P. $a = 4$, $r = 0.10$

$S_\infty = a/(1-r)$

$= 4/(1-0.10) = 4.4444444$

(66) $3, -9, 27, -81, 243$ ------ Find t_{10}, S_{12}

$\Longrightarrow$ In this G.P. $a = 3$, $r = -3$

$t_{10} = a \cdot r^9 = 3 \times (-3)^9 = 3 \times -19683 = -59049$

$$S_{12} = \frac{a(1-r^n)}{(1-r)} = \frac{3\left[1-(-3)^{12}\right]}{1-(-3)} = \frac{3(1-531441)}{4}$$

$= -3,98,580$

(67) If a, b, c are in G.P. then $\log a, \log b, \log c$ are in :

~~(a)~~ AP (b) GP (c) HP (d) None

$\Longrightarrow$ As a, b, c are in GP

$b^2 = ac$

$\log b^2 = \log(ac)$

$2\log b = \log a + \log c$

$\log b + \log b = \log a + \log c$

$\log b - \log a = \log c - \log b$

$\therefore \log a, \log b, \log c$ are in A.P.

(68) If $a, b, c, d, e, f, g, h, i$ are in A.P. then $(c-a) = ?$

(a) $d-b$ (b) $g-e$ (c) $i-g$ ~~(d)~~ All of these

(69) $\sum_{i=3}^{i=8} (2i+3) = ?$

$= [2(3)+3] + [2(4)+3] + [2(5)+3] + [2(6)+3] + [2(7)+3] + [2(8)+3]$

$= 9 + 11 + 13 + 15 + 17 + 19 = 84$

(70) For AP $S_p = 8p^2 - 50p$ Find t_p

$\Longrightarrow$ $S_p = 8p^2 - 50p$

$S_1 = 8(1)^2 - 50(1) = -42$

$S_2 = 8(2)^2 - 50(2) = -68$

$a = -42$, $t_2 = -26$, $d = 16$

$t_p = a + (p-1)d$

$= -42 + (p-1)16$

$= -42 + 16p - 16$

$= 16p - 58$

(71) For AP $a = 30$, $d = -1/5$

Find S_{150}, t_{200}

$\Longrightarrow$

$S_{150} = \frac{150}{2}\left[60 + \left(149 \times -\frac{1}{5}\right)\right]$

$= 75\left(60 - \frac{149}{5}\right) = 75(60 - 29.80)$

$= 2265$

$t_{200} = a + 199d$

$= 30 + (199 \times -1/5)$

$= 30 - 39.80$

$= -9.80$

Test on First 2 chapters

Q1) Two numbers are in the ratio 2 : 3 and the difference of their squares is 320. The numbers are:

(a) 12, 18 ~~(b)~~ 16, 24
(c) 14, 21 (d) None.

$\Rightarrow$ 2 number be $2x$ & $3x$

$$9x^2 - 4x^2 = 320$$
$$5x^2 = 320$$
$$x^2 = 64$$
$$x = 8$$

$\therefore$ Numbers are

$2x = 16$

$3x = 24$

Q2) An alloy is to contain copper and zinc in the ratio 9 : 4. The zinc required to melt with 24 kg of copper is :

~~(a)~~ $10\frac{2}{3}$kg (b) $10\frac{1}{3}$kg
(c) $9\frac{2}{3}$kg (d) 9kg

copper	zinc
9	4
24kgs	?

$$? = \frac{24 \times 4}{9} = 10.66666 \text{ kgs}$$
$$= 10\tfrac{2}{3} \text{ kgs}$$

Q3) $7 \log\left(\frac{16}{15}\right) + 5 \log\left(\frac{25}{24}\right) + 3 \log\left(\frac{81}{80}\right)$ is equal to :

(a) 0 (b) 1
~~(c)~~ log 2 (d) log 3

$$= \log\left(\frac{16^7}{15^7}\right) + \log\left(\frac{25^5}{24^5}\right) + \log\left(\frac{81^3}{80^3}\right)$$
$$= \log\left(\frac{16^7}{15^7} \times \frac{25^5}{24^5} \times \frac{81^3}{80^3}\right)$$
$$= \log\left(\frac{2^{28} \times \cancel{5^{10}} \times \cancel{3^{12}}}{\cancel{5^7} \times \cancel{3^7} \times 2^{15} \times \cancel{3^5} \times \cancel{5^3} \times 2^3 \times 2^9}\right)$$

$$= \log\left(\frac{2^{28}}{2^{27}}\right)$$
$$= \log 2^{28-27}$$
$$= \log 2^1$$
$$= \log 2$$

Q4) A box contains ₹ 56 in the form of coins of one rupee, 50 paise and 25 paise. The number of 50 paise coin is double the number of 25 paise coins and four times the numbers of one rupee coins. The numbers of 50 paise coins in the box is :

~~(a)~~ 64 (b) 32
(c) 16 (d) 14

₹		No. of coins		value
1	$\times$	x	=	x
0.50	$\times$	$4x$	=	$2x$
0.25	$\times$	$2x$	=	$0.50x$
				$3.50x$

$$3.50x = 56$$
$$\boxed{x = 16}$$

$\therefore$ No. of 50 paise coins
$= 4x = 4 \times 16 = 64$

$$\log(2a-3b) = \log(a/b)$$
$$2a-3b = \frac{a}{b}$$
$$2ab-3b^2 = a$$
$$2ab-a = 3b^2$$
$$a(2b-1) = 3b^2$$
$$a = \left(\frac{3b^2}{2b-1}\right)$$

Q5) If log (2a – 3b) = log a – log b, then a = :

~~(a)~~ $\frac{3b^2}{2b-1}$ (b) $\frac{3b}{2b-1}$
(c) $\frac{b^2}{2b+1}$ (d) $\frac{3b^2}{2b+1}$

Q6) The difference between the simple and compound interest on a certain sum for 3 year at 5% p.a. is ₹ 228.75. The compound interest on the sum for 2 years at 5% p.a. is :

(a) ₹ 3,175 ~~(b)~~ ₹ 3,075
(c) ₹ 3,275 (d) ₹ 2,975.

$$P\left[(1.05)^3-1\right] - P\times 3\times 5\% = 228.75$$
$$0.157625P - 0.15P = 228.75$$
$$0.007625P = 228.75$$
$$P = 30{,}000$$

CI
$$= 30{,}000\,(1.05^2-1)$$
$$= 3075$$

Q7) In what time will ₹ 3,90,625 amount to ₹ 4,56,976 at 8% per annum, when the interest is compounded semi-annually?

[Given : $(1.04)^4 = 1.16986$]

~~(a)~~ 2 years (b) 4 years
(c) 5 years (d) 7 years

$$4{,}56{,}976 = 3{,}90{,}625 \times \left(1+\frac{0.08}{2}\right)^{2n}$$
$$1.16986 = 1.04^{2n}$$
$$1.04^4 = 1.04^{2n}$$
$$\therefore\ 2n = 4$$
$$n = 2 \text{ years}$$

Q8) The annual birth and death rates per 1000 are 39.4 and 19.4 respectively. The number of years in which the population will be doubled assuming there is no immigration or emigration is :

~~(a)~~ 35 years (b) 30 years
(c) 25 years (d) None of thee

$$1000 + 39.4 - 19.4 = 1020$$
Growth rate $= 2\%$

$$A = P(1+r)^n$$
$$2\cancel{P} = \cancel{P}(1.02)^n$$

$$1.02^n = 2$$
$$1.02^n = 1.02^{35}$$
$$n = 35 \text{ years}$$

Q9) The effective rate equivalent to nominal rate of 6% compounded monthly is :

(a) 6.05 ~~(b)~~ 6.16

(c) 6.26 (d) 6.07

$$\text{Effective rate} = \left(1+\frac{0.06}{12}\right)^{12} - 1$$

$$= 1.005^{12} - 1$$

$$= 6.16778\%$$

Q10) If the difference between simple interest and compound interest is ₹ 11 at the rate of 10% for two years, then find the sum.

(a) ₹ 1,200 ~~(b)~~ ₹ 1,100

(c) ₹ 1,000 (d) None of these

$$P\left[(1.10)^2 - 1\right] - P \times 2 \times 10\% = 11$$

$$0.21P - 0.20P = 11$$

$$0.01P = 11$$

$$P = 1100$$

Q11) At what % rate of compound interest (C.I) will a sum of money become 16 times in four years, if interest is being calculated compounding annually:

~~(a)~~ r = 100% (b) r = 10%

(c) r = 200% (d) r = 20%

$$A = P(1+r)^n$$

$$16\cancel{P} = \cancel{P}(1+r)^4$$

$$(1+r)^4 = 2^4$$

$$1+r = 2$$

$$r = 1.00 = 100\%$$

Q12) The future value of an annuity of ₹ 5,000 is made annually for 8 years at interest rate of 9% compounded annually [Given that $(1.09)^8 = 1.99256$] is________

~~(a)~~ ₹ 55,142.22 (b) ₹ 65,142.22

(c) ₹ 65,532.22 (d) ₹ 57,425.22

$$\text{Future value of annuity regular} = 5000 \times \left(\frac{1.09^8 - 1}{0.09}\right)$$

$$= 55,142.2222$$

Q13) The incomes of A and B are in the ratio 3 : 2 and their expenditures in the ratio 5 : 3. If each saves ₹ 1,500, then B's income is :

~~(a)~~ ₹ 6,000 (b) ₹ 4,500

(c) ₹ 3,000 (d) ₹ 7,500

Incomes

$$\frac{3x - 1500}{2x - 1500} = \frac{5}{3}$$

$3x = 9000$

$2x = 6000$

$$9x - 4500 = 10x - 7500$$

$$3000 = x$$

Q14) In 40 litres mixture of glycerine and water, the ratio of glycerine and water is 3:1. The quantity of water added in the mixture in order to make this ratio 2:1 is:

(a) 15 litres (b) 10 litres

(c) 8 litres ~~(d)~~ 5 litres.

Gly : 30

water : $10 + x$

$$\frac{30}{10+x} = \frac{2}{1}$$

$$20 + 2x = 30$$

$$2x = 10$$

$x = 5$ litres

Q15) Fourth proportional to x, 2x, (x+1) is:

(a) (x + 2) (b) (x – 2)

~~(c)~~ (2x + 2) (d) (2x – 2)

$$x,\ 2x,\ (x+1),\ m$$

$$xm = 2x(x+1)$$

$$m = \frac{2\cancel{x}(x+1)}{\cancel{x}} = 2x + 2$$

Q16) The recurring decimal 2.7777........ can be expressed as:

(a) 24/9 (b) 22/9

(c) 26/9 ~~(d)~~ 25/9

Q17) If $\log_2 x + \log_4 x = 6$, then the Value of x is :

~~(a)~~ 16 (b) 32
(c) 64 (d) 128

$$\frac{2\operatorname{Log}x}{2\operatorname{Log}2} + \frac{\operatorname{Log}x}{2\operatorname{Log}2} = 6$$

$$\frac{3\operatorname{Log}x}{2\operatorname{Log}2} = 6$$

$3\operatorname{Log}x = 12\operatorname{Log}2$

$\operatorname{Log}x = 4\operatorname{Log}2$

$\operatorname{Log}x = \operatorname{Log}2^4$

$\operatorname{Log}x = \operatorname{Log}16$

$x = 16$

Q18) Which of the numbers are not in proportion ?

~~(a)~~ 6, 8, 5, 7 (b) 7, 3, 14, 6
(c) 18, 27, 12, 18 (d) 8, 6, 12, 9

Q19) If x : y = 2:3, then (5x+2y):(3x-y)= ____

(a) 19 : 3 ~~(b)~~ 16 : 3
(c) 7 : 2 (d) 7 : 3

$x : y = 2 : 3$

$x = 2k$

$y = 3k$

$$\frac{5x+2y}{3x-y}$$

$$= \frac{5(2k)+2(3k)}{3(2k)-3k} = \frac{16k}{3k} = 16:3$$

Q20) By mistake a clerk, calculated the simple interest on principal for 5 months at 6.5% p.a. instead of 6 months at 5.5% p.a. If the error in calculation was ₹ 25.40. The original sum of principal was_____.

(a) ₹ 60,690 ~~(b)~~ ₹ 60,960
(c) ₹ 90,660 (d) ₹ 90,690

$P \times \frac{5}{12} \times 6.50\% = 0.0270833333\,P$

$P \times \frac{6}{12} \times 5.50\% = 0.0275\,P$

$0.000416666666\,P = 25.40$

$P = 60,960$

Q21) How much investment is required to yield an Annual income of ₹ 420 at 7% p.a. Simple interest.

~~(a)~~ ₹ 6,000 (b) ₹ 6,420
(c) ₹ 5,580 (d) ₹ 5,000

$$P \cdot n \cdot r = 420$$
$$P \times 1 \times 0.07 = 420$$
$$P = 6000$$

Q22) A sum of money compounded annually becomes ₹ 1,140 in two years and ₹ 1,710 in three years.
Find the rate of interest per annum.

(a) 30% (b) 40%
~~(c)~~ 50% (d) 60%

$$1140 \times (1+r) = 1710$$
$$1+r = 1.50$$
$$r = 0.50$$

Q23) A person lends ₹ 6,000 for 4 years and ₹ 8,000 for 3 years at simple interest. If he gets ₹ 2,400 as total interest, the rate of interest is:

~~(a)~~ 5% (b) 4%
(c) 6% (d) 7%

$$6000 \times 4 \times r = 24000r$$
$$8000 \times 3 \times r = 24{,}000r$$
$$48{,}000r = 2400$$
$$r = 0.05$$

Q24) How much amount is required to be invested every year so as to accumulate ₹ 3,00,000 at the end of 10 years, if interest is compounded annually at 10%?

~~(a)~~ ₹ 18,823.65 (b) ₹ 18
(c) ₹ 18,828.65 (d) ₹ 18,882.65

$$3{,}00{,}000 = P.A. \times \left(\frac{1.10^{10} - 1}{0.10}\right)$$
$$P.A. = 18{,}823.618$$

Q25) A certain sum of money Q was deposited for 5 year and 4 months at 4.5% simple interest and amounted to ₹ 248, then the value of Q is

~~(a)~~ ₹ 200 (b) ₹ 210

(c) ₹ 220 (d) ₹ 240

$$248 = P(1+nr)$$

$$248 = Q\left[1+(5.3333333 \times 0.045)\right]$$

$$248 = Q \times 1.24$$

$$Q = 200$$

Q26) A dealer mixes rice costing ₹ 13.84 per Kg. with rice costing ₹ 15.54 and sells the mixture at ₹ 17.60 per Kg. So, he earns a profit of 14.6% on his sale price. The proportion in which he mixes the two qualities of rice is:

(a) 3 : 7 (b) 5 : 7

(c) 7 : 9 (d) 9 : 11

selling price per kg = 17.60

profit per kg = 2.5696

cost per kg = 15.0304

x kgs	y kgs
↓	↓
13.84 per kg	15.54 per kg

Total cost of Rice

$$13.84x + 15.54y = 15.0304(x+y)$$

$$13.84x + 15.54y = 15.0304x + 15.0304y$$

$$0.5096y = 1.1904x$$

$$\frac{x}{y} = \frac{0.5096}{1.1904} = \frac{0.51}{1.19} = \frac{51}{119} = \frac{3}{7}$$

Q27) What number must be added to each of the numbers 10, 18, 22, 38 to make the numbers is proportion?

~~(a)~~ 2 (b) 4

(c) 8 (d) None of these.

12, 20, 24, 40 are in proportion

Q28) If log 2 = 0.3010 and log 3 = 0.4771, then the value of log 24 is:

(a) 1.0791 (b) 1.7323

~~(c)~~ 1.3801 (d) 1.8301

$$\text{Log}(24) = \text{Log}(2 \times 2 \times 2 \times 3)$$

$$= \text{Log}2 + \text{Log}2 + \text{Log}2 + \text{Log}3$$

$$= 3\,\text{Log}2 + \text{Log}3$$

$$= (3 \times 0.3010) + 0.4771$$

$$= 1.3801$$

Q29) There are total 23 coins of ₹ 1, ₹ 2 and ₹ 5 in a bag. If their value is ₹ 43 and the ratio of coins of ₹ 1 and ₹ 2 is 3:2. Then the number of coins of ₹ 1 is:

~~(a)~~ 12 (b) 5
(c) 10 (d) 14

₹	No. of coins	value
1	$3x$	$3x$
2	$2x$	$4x$
5	$(23-5x)$	$115-25x$

$$3x+4x+115-25x=43$$
$$-18x = 43-115$$
$$-18x = -72$$
$$x=4$$

Q30) The mean proportional between 24 and 54 is:

(a) 33 (b) 34
(c) 35 ~~(d)~~ 36

$$24, m, 54$$
$$m^2 = 24 \times 54 = 1296$$
$$m = 36$$